1.3 — Budget Constraint

ECON 306 • Microeconomic Analysis • Fall 2020

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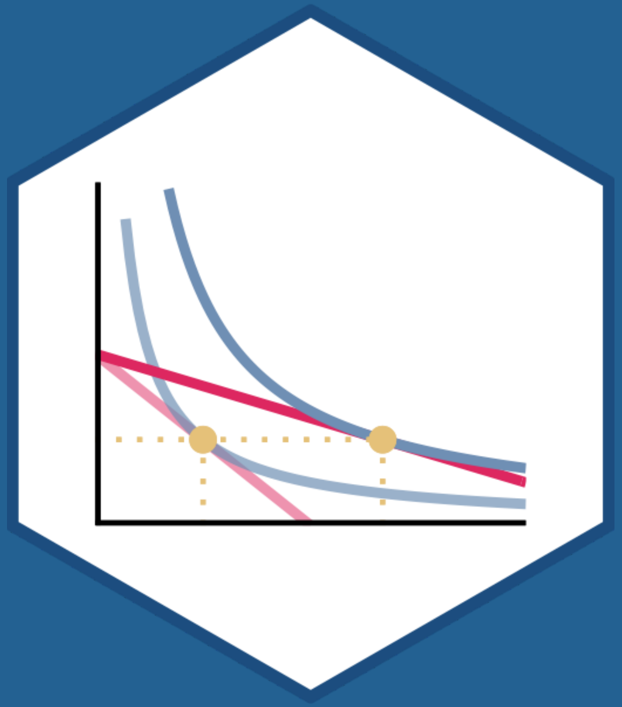

Outline

The Two Major Models of Economics as a "Science"

Optimization

- Agents have **objectives** they value
- Agents face **constraints**
- Make **tradeoffs** to maximize objectives within constraints

Equilibrium

- Agents **compete** with others over **scarce** resources
- Agents **adjust** behaviors based on prices
- **Stable outcomes** when adjustments stop

Rational Choice Theory

Consumer Behavior

- How do people decide:
  - which products to buy
  - which activities to dedicate their time to
  - how to save or invest/plan for the future
- A model of behavior we can extend to most scenarios
- Answers to these questions are building blocks for **demand curves**

Rational Choice Theory: Beyond Consumers

- *Everyone* is a consumer
    - "Goods and services" isn't just food, clothing, etc, but *anything* that you value!
- Consumers making purchasing decisions will be our paradigmatic *example*
    - But we are really talking about how **individuals** make choices in almost *any* context!

Constrained Optimization

Constrained Optimization I

- We model most situations as a **constrained optimization problem**:

- People **optimize**: make tradeoffs to achieve their **objective** *as best as they can*

- Subject to **constraints**: limited resources (income, time, attention, etc)

Constrained Optimization II

- One of the most generally useful mathematical models

- *Endless applications*: how we model nearly every decision-maker

> consumer, business firm, politician, judge, bureaucrat, voter, dictator, pirate, drug cartel, drug addict, parent, child, etc

- **Key economic skill: recognizing how to apply the model to a situation**

Constrained Optimization III

- All constrained optimization models have three moving parts:

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1. **Choose:** **< some alternative >**

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2. **In order to maximize:** < some objective >

Constrained Optimization III

- All constrained optimization models have three moving parts:

1. **Choose:** < some alternative >

2. **In order to maximize:** < some objective >

3. **Subject to:** < some constraints >

Constrained Optimization: Example I

Example: A Hood student picking courses hoping to achieve the highest GPA while getting an Econ major.

1. **Choose:**

2. **In order to maximize:**

3. **Subject to:**

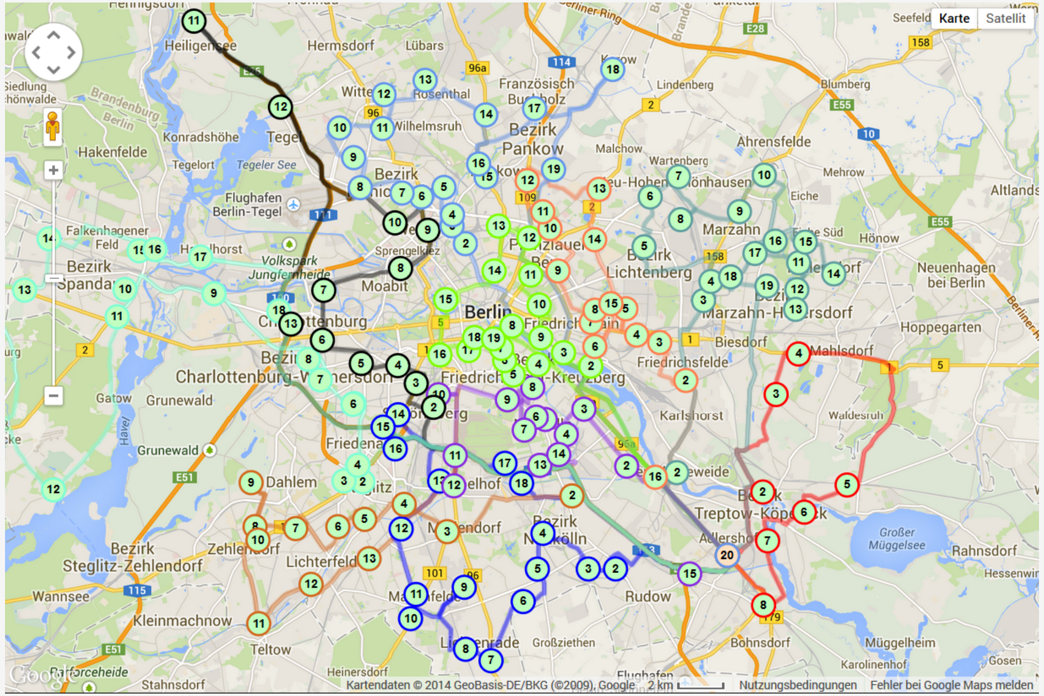
Constrained Optimization: Example II

Example: How should FedEx plan its delivery route?

1. **Choose:**

2. **In order to maximize:**

3. **Subject to:**



Constrained Optimization: Example III

Example: The U.S. government wants to remain economically competitive but reduce emissions by 25%.

1. **Choose:**

2. **In order to maximize:**

3. **Subject to:**

Constrained Optimization: Example IV

Example: How do elected officials make decisions in politics?

1. **Choose:**
2. **In order to maximize:**
3. **Subject to:**

The Consumer's Problem

- The **consumer's constrained optimization problem** is:

The Consumer's Problem

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1. **Choose:** < a consumption bundle >

The Consumer's Problem

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The Consumer's Problem

- The consumer's constrained optimization problem is:

1. **Choose:** < a consumption bundle >

2. **In order to maximize:** < utility >

3. **Subject to:** < income and market prices >

Consumer Behavior: Basic Framework

Consumption Bundles

- Imagine a (very strange) supermarket sells xylophones (x) and yams (y)

- Your choices: amounts of x, y to buy as a **bundle**

Consumption Bundles

- Represent bundles as a vector:

$$a = \begin{pmatrix} x \\ y \end{pmatrix}$$

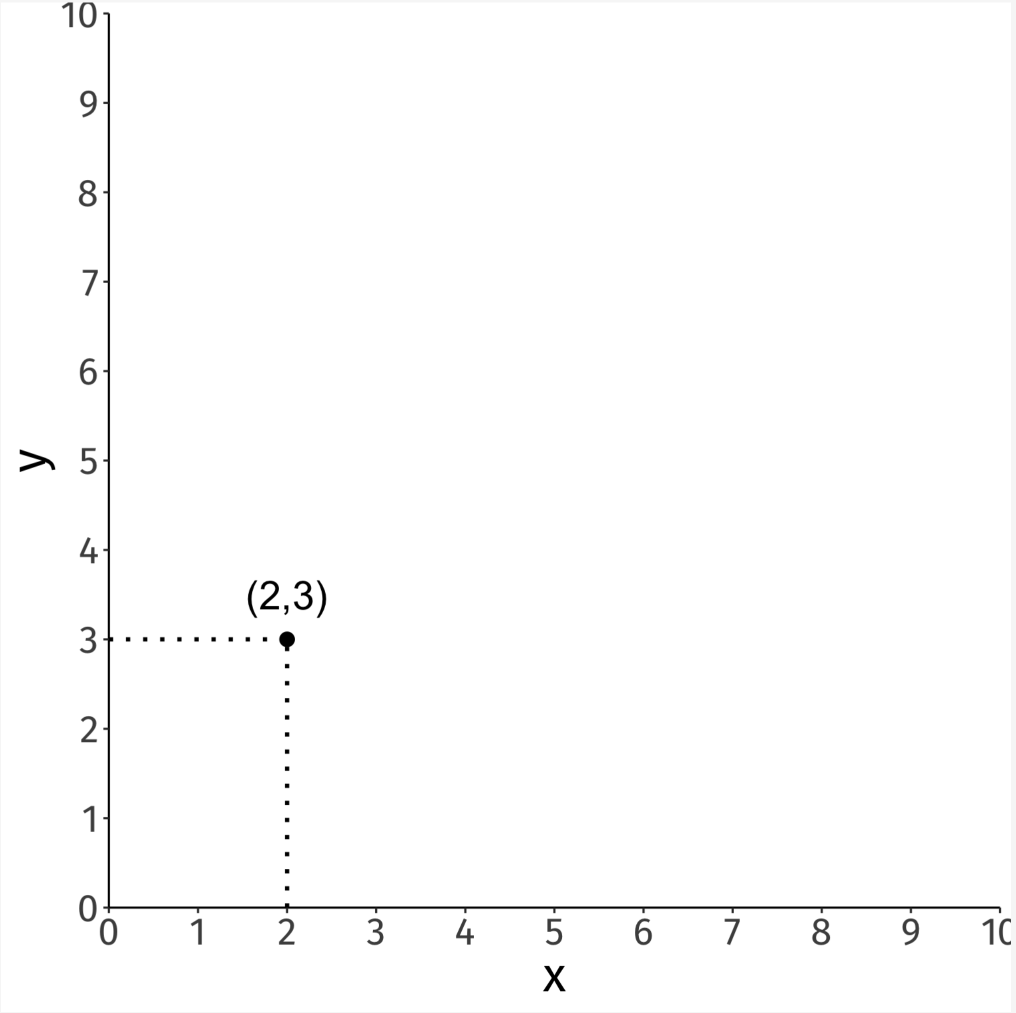
Examples:

$$a = \begin{pmatrix} 4 \\ 12 \end{pmatrix};\ b = \begin{pmatrix} 6 \\ 12 \end{pmatrix};\ c = \begin{pmatrix} 21 \\ 0 \end{pmatrix}$$

Consumption Bundles: Graphically

- We can represent bundles graphically

- We'll stick with 2 goods (x, y) in 2-dimensions



The Budget Constraint

Affordability

- If you had \$100 to spend, what bundles of goods $\{x, y\}$ would you buy?
- Only those bundles that are **affordable**
- Denote prices of each good as $\{p_x, p_y\}$
- Let m be the amount of income a consumer has

Affordability

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- Only those bundles that are **affordable**
- Denote prices of each good as $\{p_x, p_y\}$
- Let m be the amount of income a consumer has
- A bundle $\{x, y\}$ is **affordable** at given prices $\{p_x, p_y\}$ when:

$$p_x x + p_y y \leq m$$

The Budget Set

- The set of *all* affordable bundles that a consumer can choose is called the **budget set** or **choice set**

$$p_x x + p_y y \leq m$$

The Budget Set & the Budget Constraint

- The set of *all* affordable bundles that a consumer can choose is called the **budget set** or **choice set**

$$p_x x + p_y y \leq m$$

- The **budget *constraint*** is the set of all bundles that spend *all income* m:†

$$p_x x + p_y y = m$$

† Note the difference (the in/equality), budget *constraint* is the **subset** of the *budget set* that *spends all income*.

The Budget Constraint, Graphically

- For 2 goods, (x, y)

$$p_x x + p_y y = m$$

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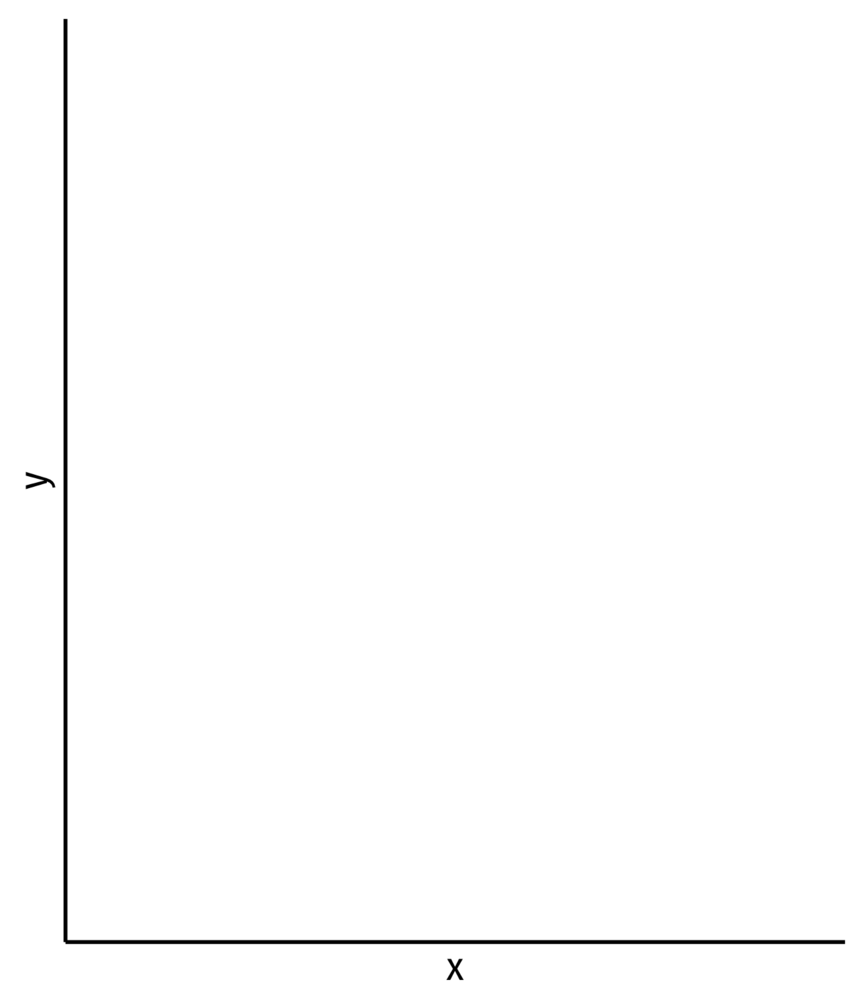
- For 2 goods, (x, y)

$$p_x x + p_y y = m$$

- Solve for y to graph

$$y = \frac{m}{p_y} - \frac{p_x}{p_y} x$$



The Budget Constraint, Graphically

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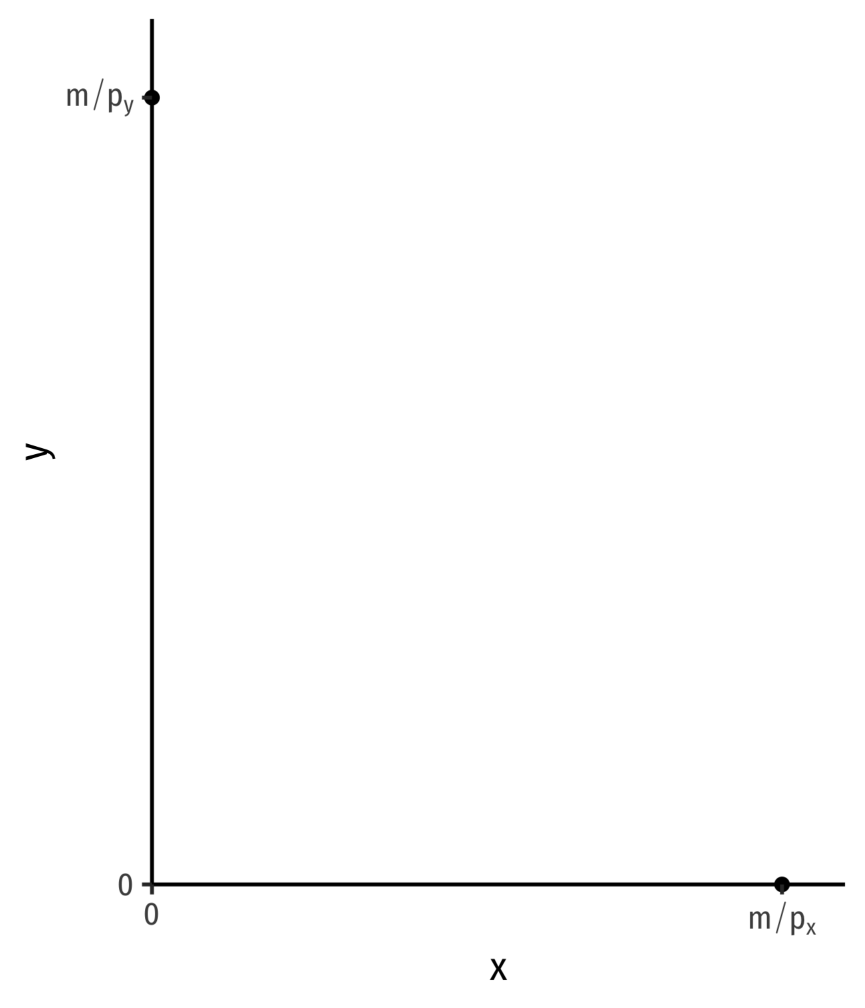
$$y = \frac{m}{p_y} - \frac{p_x}{p_y}x$$

- y-intercept: $\frac{m}{p_y}$
- x-intercept: $\frac{m}{p_x}$



The Budget Constraint, Graphically

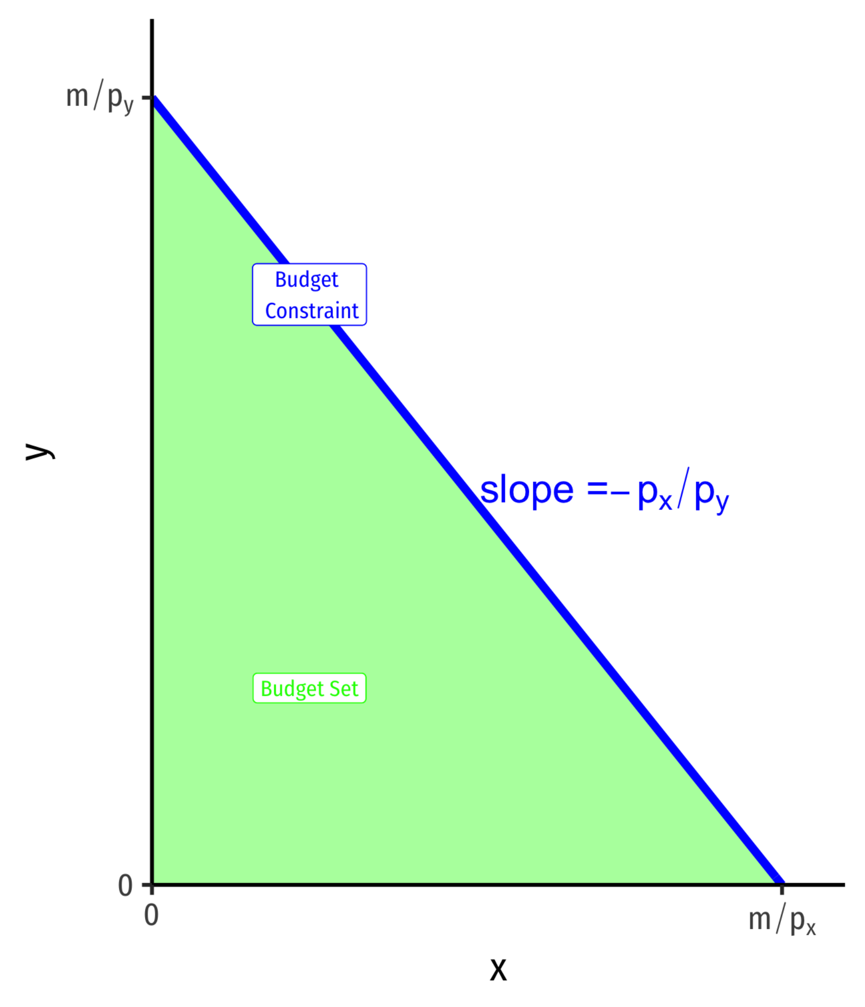
- For 2 goods, (x, y)

$$p_x x + p_y y = m$$

- Solve for y to graph

$$y = \frac{m}{p_y} - \frac{p_x}{p_y} x$$

- y-intercept: $\frac{m}{p_y}$
- x-intercept: $\frac{m}{p_x}$
- slope: $\frac{p_x}{p_y}$



The Budget Constraint: Example

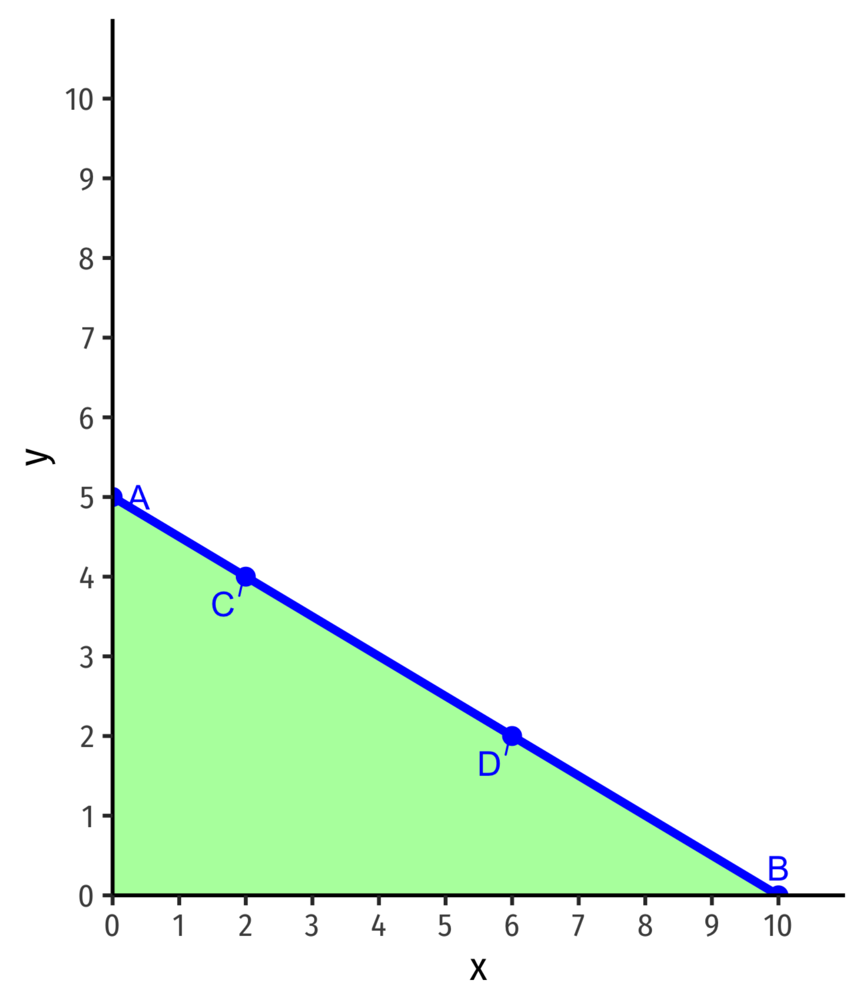
Example: Suppose you have an income of \$50 to spend on lattes (l) and burritos (b). The price of lattes is \$5 and the price of burritos is \$10. Let l be on the horizontal axis and b be on the vertical axis.

1. Write an equation for the budget constraint (in graphable form).

2. Graph the budget constraint.

Interpreting the Budget Constraint

- Points **on** the line spend all income
  - A: $\$5(0x) + \$10(5y) = \$50$
  - B: $\$5(10x) + \$10(0y) = \$50$
  - C: $\$5(2x) + \$10(4y) = \$50$
  - D: $\$5(6x) + \$10(2y) = \$50$



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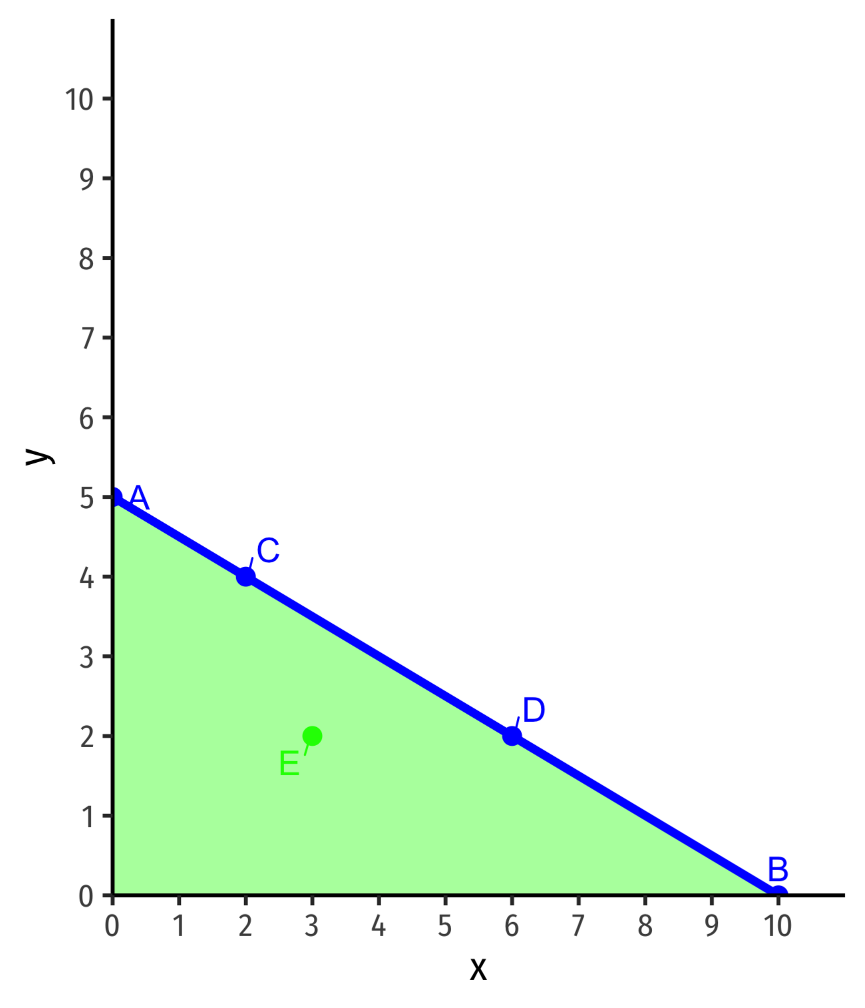
- Points beneath the line are affordable but don't use all income

    - E: $\$5(3x) + \$10(2y) = \$35$



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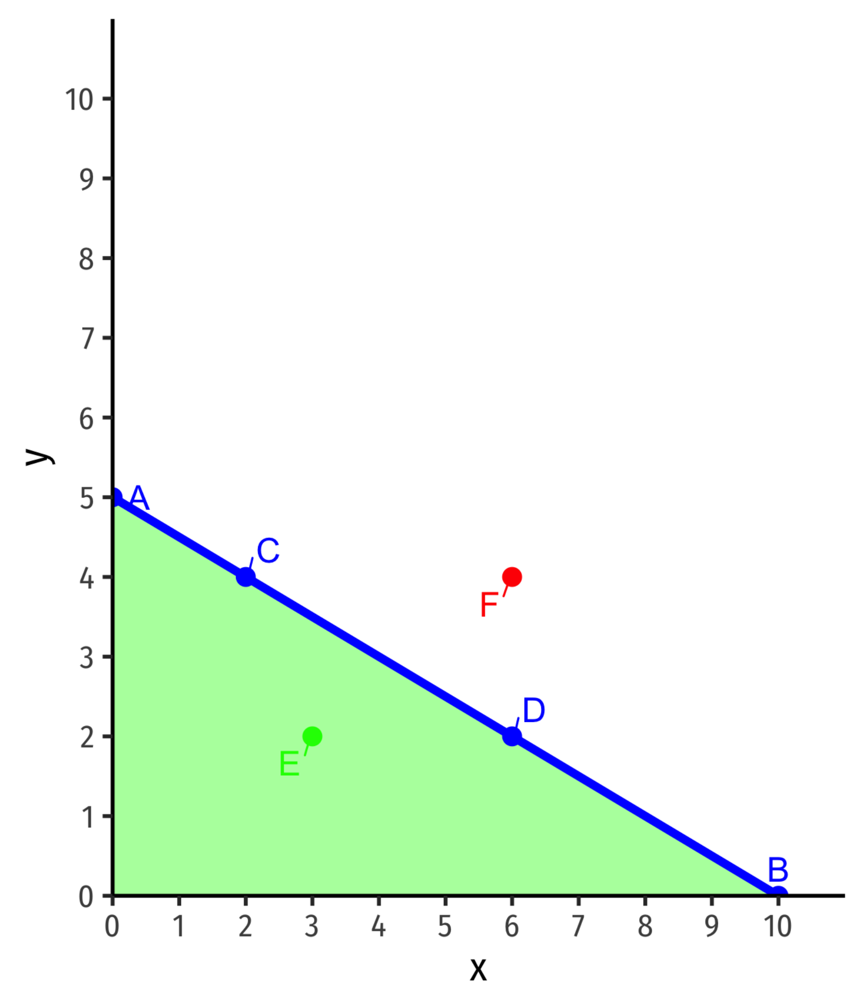
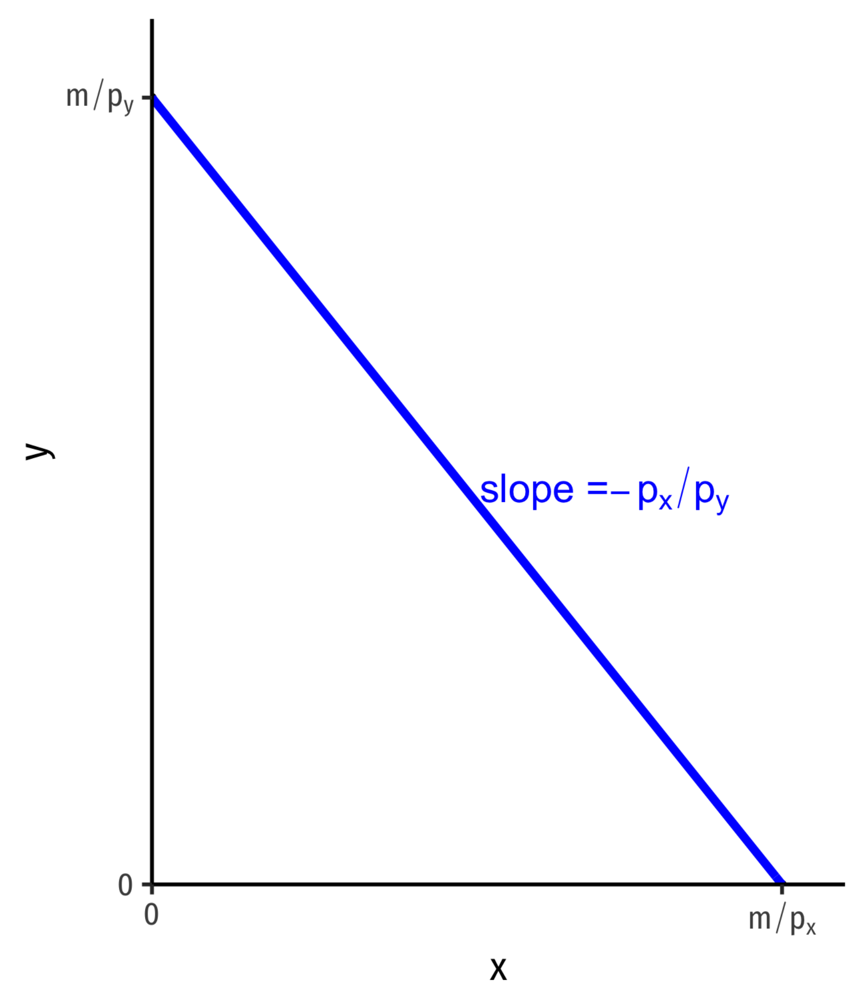
  - C: $\$5(2x) + \$10(4y) = \$50$
  - D: $\$5(6x) + \$10(2y) = \$50$
- Points **beneath** the line are **affordable** but don't use all income
  - E: $\$5(3x) + \$10(2y) = \$35$
- Points **above** the line are **unaffordable** (at current income and prices)



Interpretting the Slope

- **Slope**: market-rate of **tradeoff** between x and y
- **Relative price** of x or its **opportunity cost**:

> Consuming 1 more unit of x requires giving up $\frac{p_x}{p_y}$ units of y



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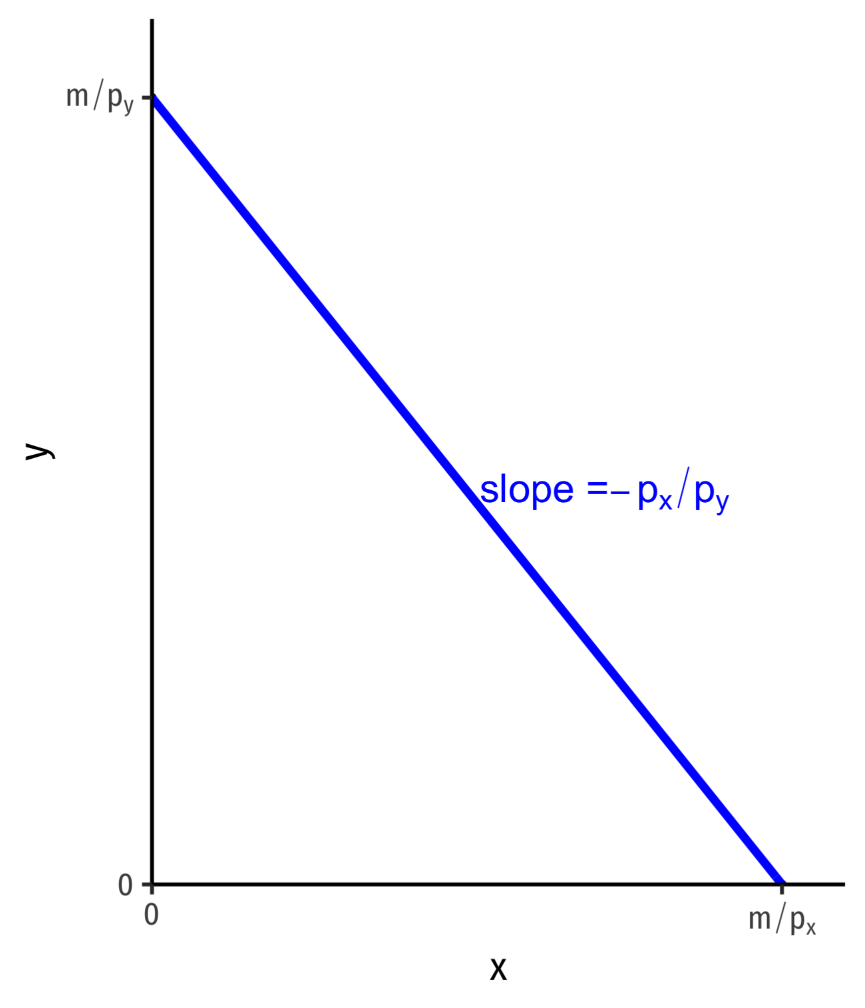
- Foreshadowing:

> Is **your** valuation of the tradeoff between x and y the same as the market rate?



Changes in Parameters

Changes in Parameters

$$m = p_x x + p_y y$$

$$y = \frac{m}{p_y} - \frac{p_x}{p_y} x$$

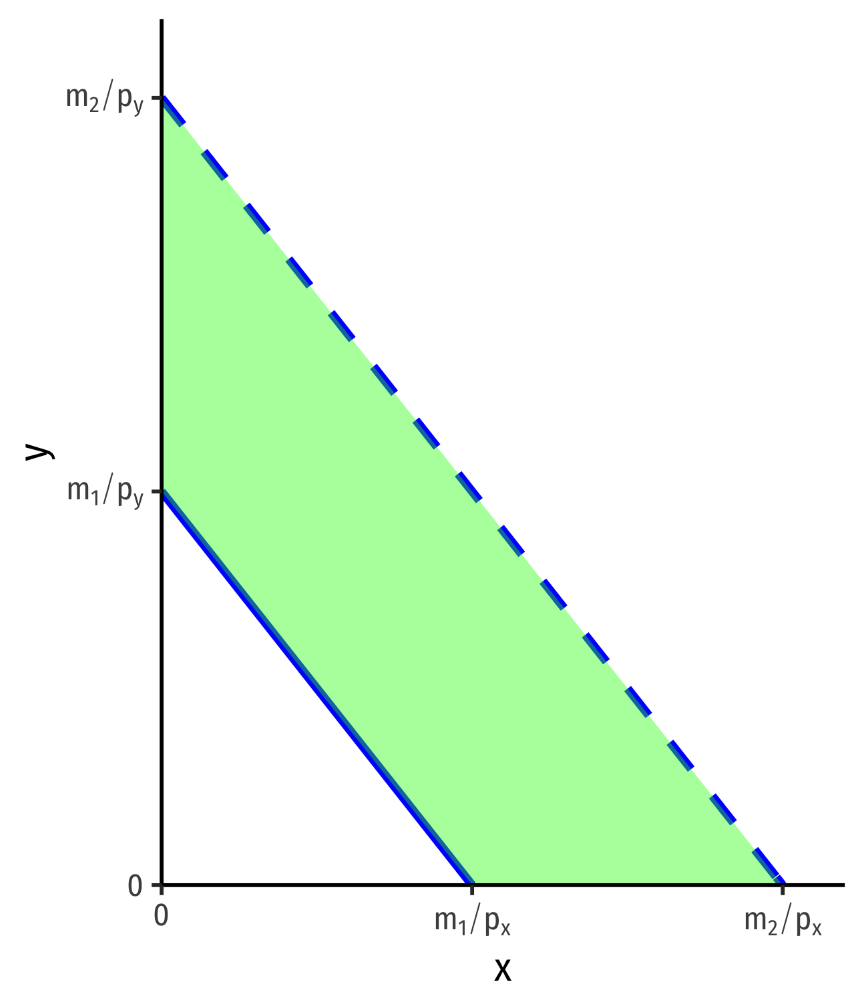
- Budget constraint is a function of specific **parameters**
  - m: income
  - p_x, p_y: market prices
- Economics: **how changes in constraints affect people's choices**

Changes in Income, m

- Changes in **income**: a *parallel* shift in budget constraint

Example: An increase in income

- Same slope (relative prices don't change!)
- **Gain of affordable bundles**



Changes in Income, m: Example

Example: Continuing the lattes and burritos example, (income is $50, lattes are $5, burritos are $10), suppose your income doubles to $100.

1. Find the equation of the new budget constraint (in graphable form).

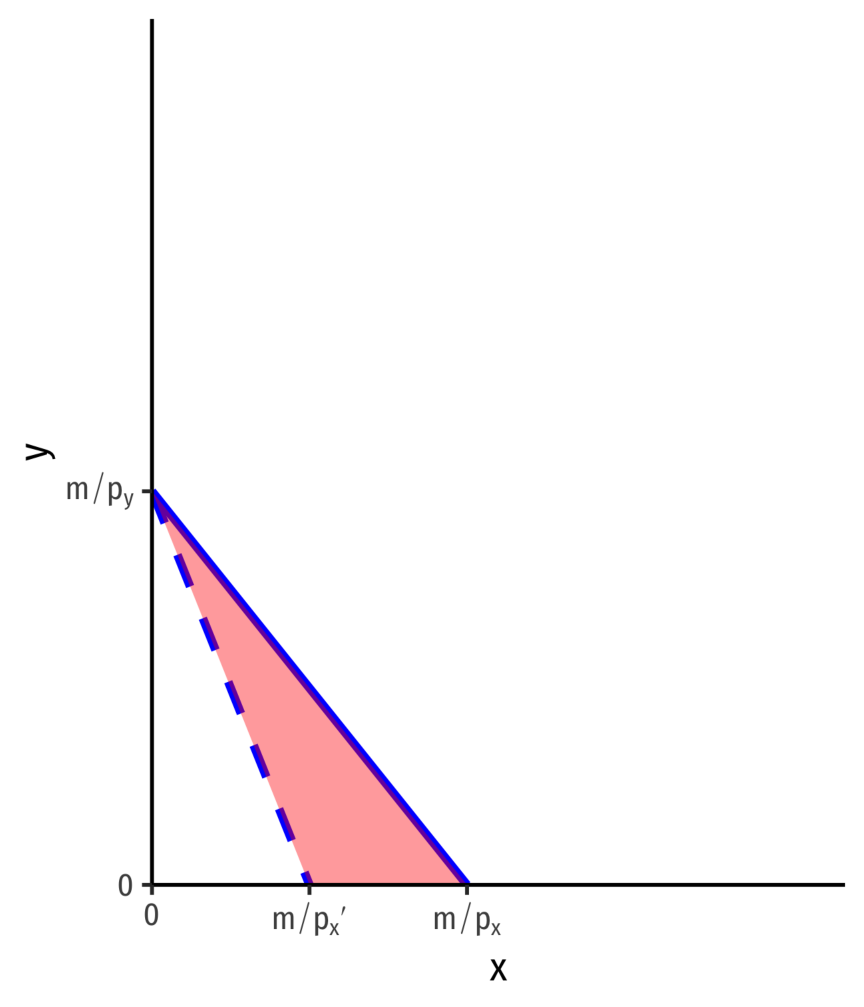
2. Graph the new budget constraint.

Changes in Relative Prices, p_x or p_y

- Changes in **relative prices**: *rotate* the budget constraint

Example: An increase in the price of x

- Slope steepens: $-\frac{p'_x}{p_y}$
- **Loss of affordable bundles**

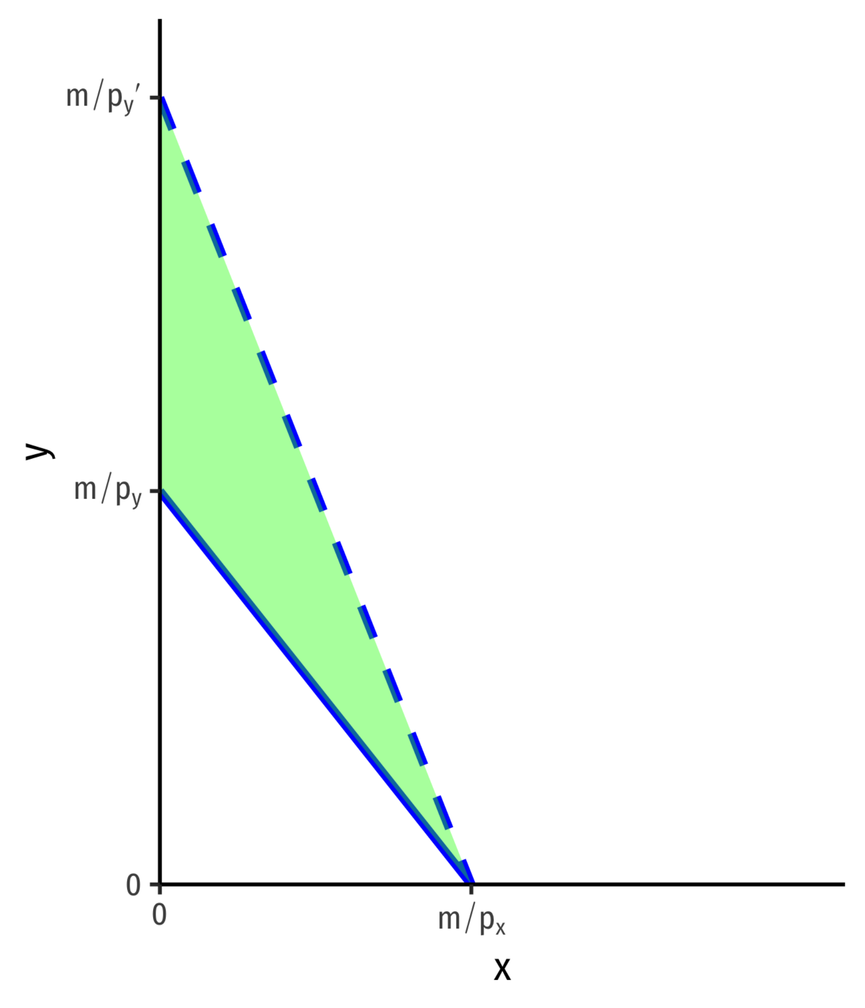


Changes in Relative Prices, p_x or p_y

- Changes in **relative prices**: *rotate* the budget constraint

Example: A decrease in the price of y

- Slope flattens: $-\frac{p_x}{p'_y}$
- **Gain of affordable bundles**



Economics is About (Changes in) *Relative* Prices

- Economics is about (changes in) *relative* prices

- Budget constraint slope is $\left(\frac{p_x}{p_y}\right)$

- Only **"real"** changes in *relative* prices (from changes in market valuations) change consumer constraints

Economics is About (Changes in) *Relative* Prices

- **"Nominal"** prices are often meaningless!

- **Example**: Imagine yourself in a strange country. All you know is that the price of bread is "6"...

Changes in Relative Prices: Example

Example: Continuing the lattes and burritos example (income is \$50, lattes are \$5, burritos are \$10).

1. Suppose the price of lattes doubles from \$5 to \$10. Find the equation of the new budget constraint and graph it.

2. Return to the original price of lattes (\$5) and suppose the price of burritos falls from \$10 to \$5. Find the equation of the new budget constraint and graph it.