1.6 — Deriving a Demand Function

ECON 306 • Microeconomic Analysis • Fall 2020
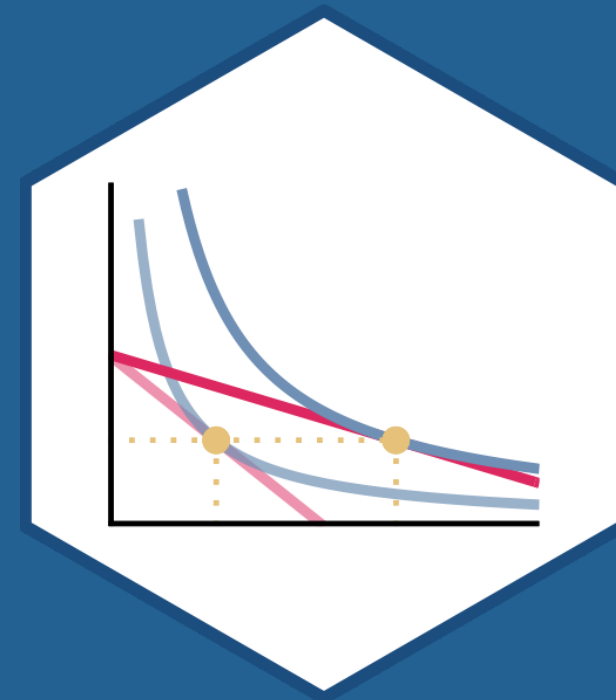
Ryan Safner

Assistant Professor of Economics
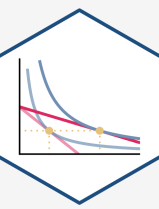
✈ safner@hood.edu

⊙ ryansafner/microF20

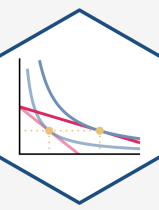🌐 microF20.classes.ryansafner.com

The Consumer's Problem: Review

- We now can explore the **dynamics** of how consumers **optimally respond to changes** in their constraints

- We know the problem is:

1. **Choose:** **< a consumption bundle >**

2. **In order to maximize:** **< utility >**

3. **Subject to:** **< income and market prices >**

A Demand Function (for Good X)

- A consumer's **demand** (for good x) depends on current prices & income:

$$q_x^D = q_x^D(m, p_x, p_y)$$

- How does **demand for x** change?

A Demand Function (for Good X)
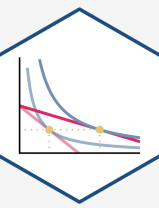
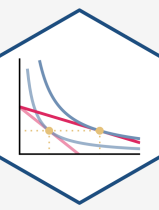- A consumer's **demand** (for good x) depends on current prices & income:

$$q_x^D = q_x^D(m, p_x, p_y)$$

- How does **demand for x** change?

1. **Income effects** $\left(\frac{\Delta q_x^D}{\Delta m} \right)$: how q_x^D changes with changes in income

A Demand Function (for Good X)

- A consumer's **demand** (for good x) depends on current prices & income:
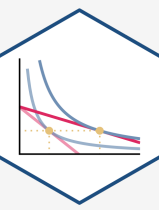
$$q_x^D = q_x^D(m, p_x, p_y)$$

- How does **demand for x** change?

1. **Income effects** $\left(\frac{\Delta q_x^D}{\Delta m}\right)$: how q_x^D changes with changes in income

2. **Cross-price effects** $\left(\frac{\Delta q_x^D}{\Delta p_y}\right)$: how q_x^D changes with changes in prices of *other* goods (e.g. y)
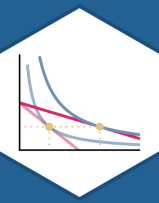
A Demand Function (for Good X)

- A consumer's **demand** (for good x) depends on current prices & income:

$$q_x^D = q_x^D(m, p_x, p_y)$$
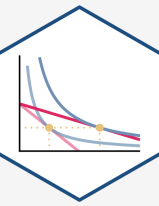
- How does **demand for x** change?

1. **Income effects** $\left(\frac{\Delta q_x^D}{\Delta m}\right)$: how q_x^D changes with changes in income

2. **Cross-price effects** $\left(\frac{\Delta q_x^D}{\Delta p_y}\right)$: how q_x^D changes with changes in prices of *other* goods (e.g. y)

3. **(Own) Price effects** $\left(\frac{\Delta q_x^D}{\Delta p_x}\right)$: how q_x^D changes with changes in price (of x)
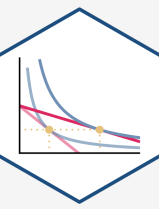
Income Effect

Income Effect

- **Income effect**: change in optimal consumption of a good associated with a change in (nominal) income, holding relative prices constant
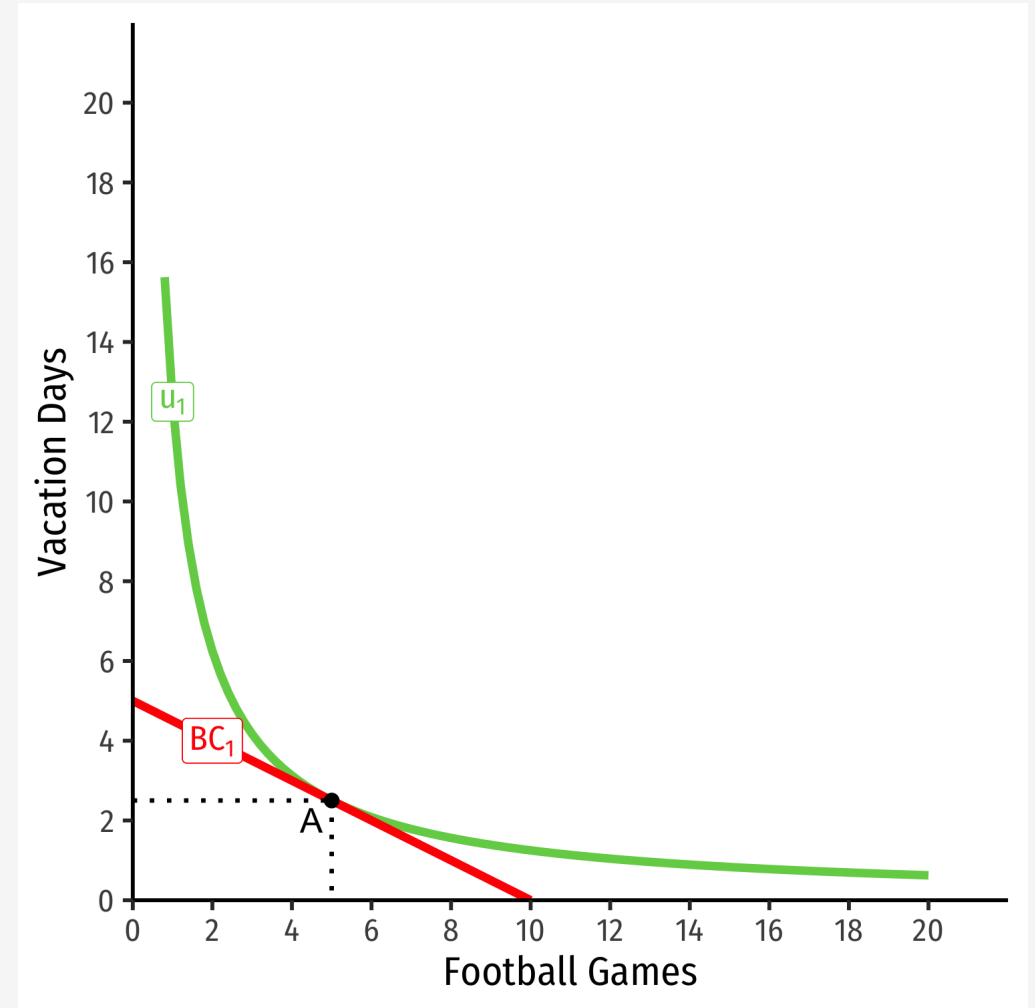
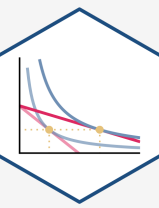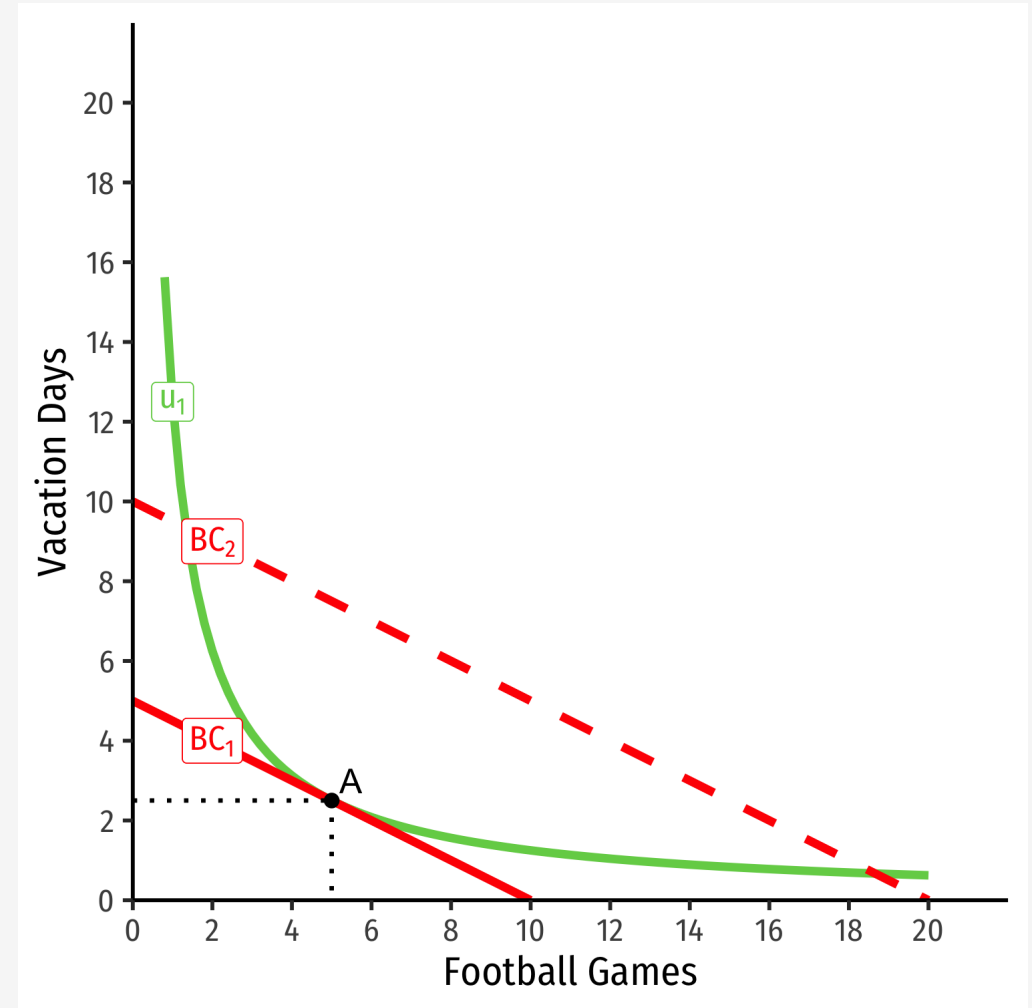$$\frac{\Delta q_D}{\Delta m} >^? < 0$$

Income Effect (Normal)
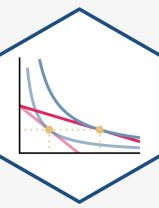
- Consider football tickets and vacation days
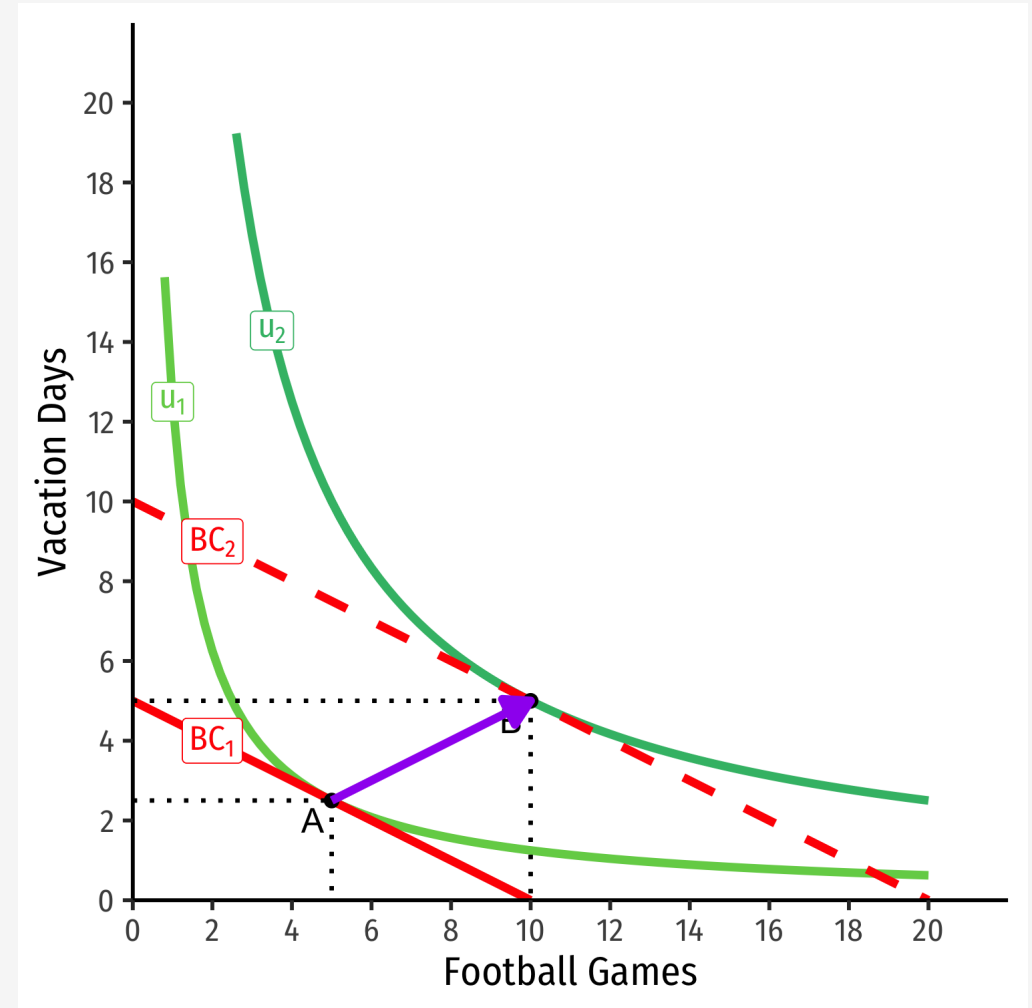
Income Effect (Normal)

- Consider football tickets and vacation days

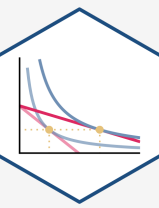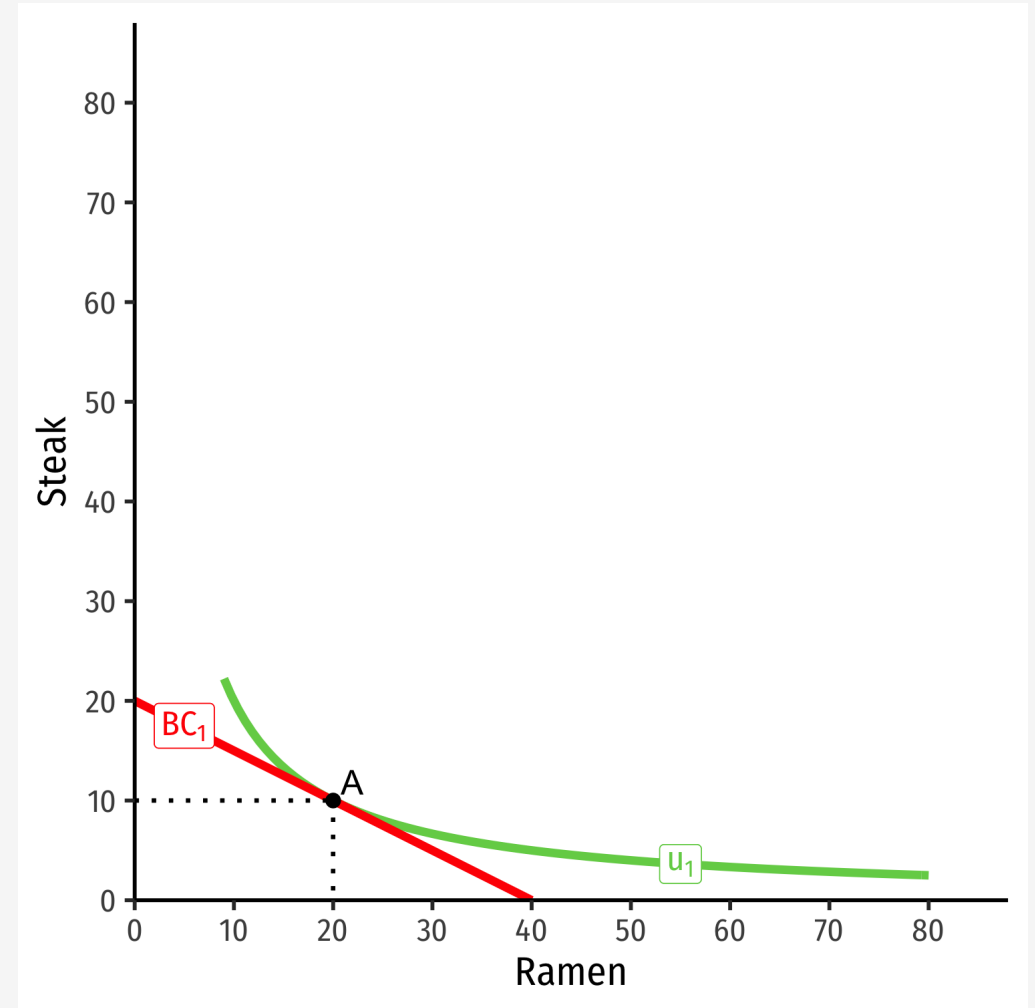- Suppose income (m) increases

Income Effect (Normal)

- Consider football tickets and vacation days

- Suppose income (m) increases

- At new optimum (B), consumes more of both
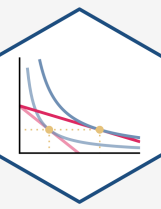
- Then both goods are **normal goods**

Income Effect (Inferior)

- Consider ramen and steak

Income Effect (Inferior)
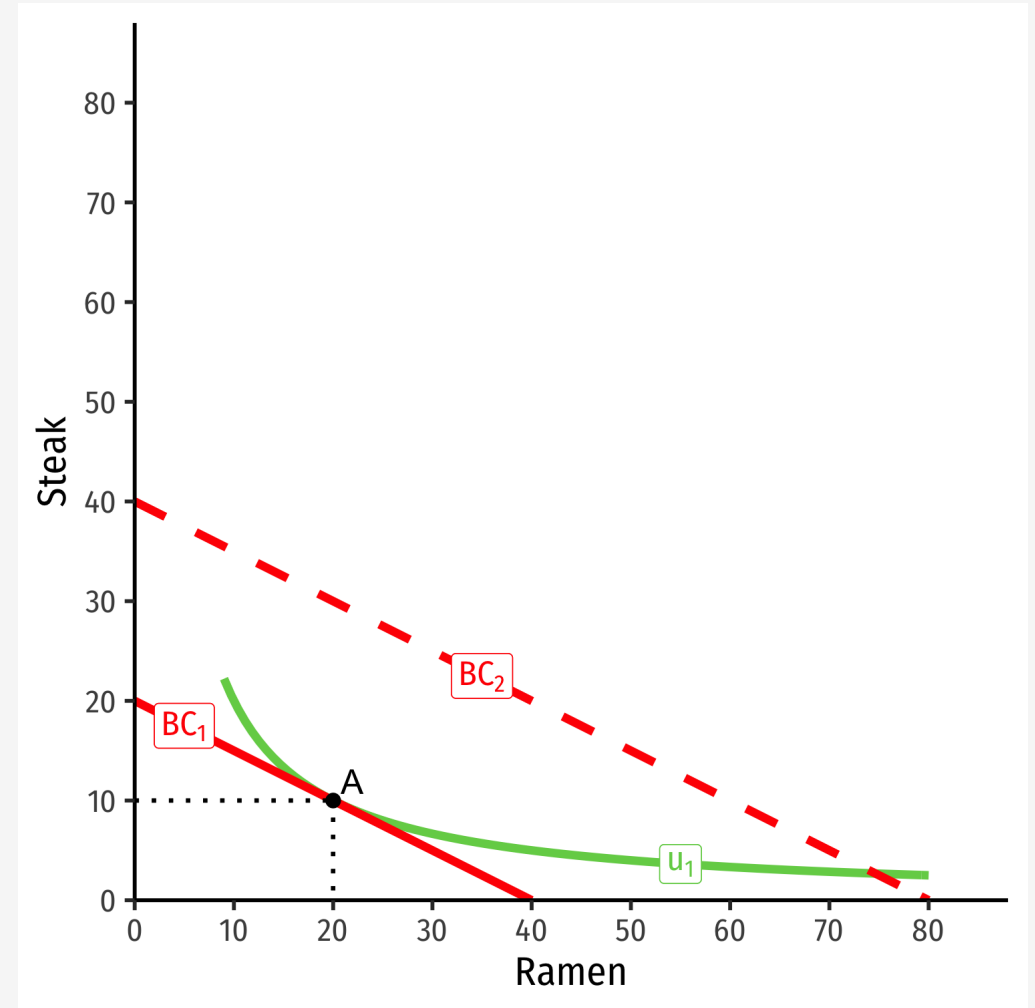
- Consider ramen and steak

- Suppose income (m) increases

Income Effect (Inferior)
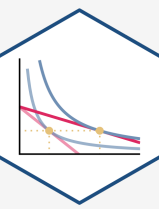
- Consider ramen and steak

- Suppose income (m) increases

- At new optimum (B), consumes more steak, less ramen

- Steak is a **normal good**, ramen is an **inferior good**
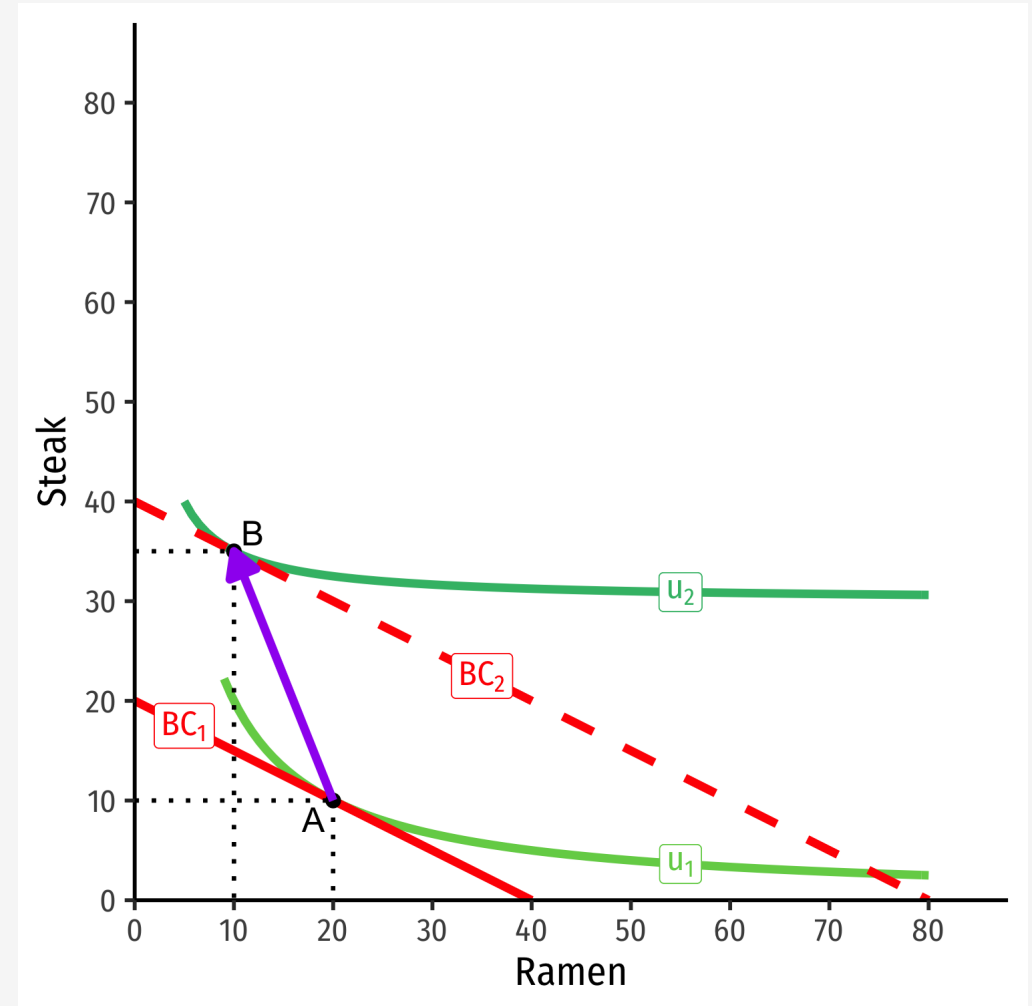
Income Effect

$$\frac{\Delta q_D}{\Delta m} >^? < 0$$

- **Normal goods**: consumption increases with more income (and vice versa)

- **Inferior goods**: consumption decreases with more income (and vice versa)
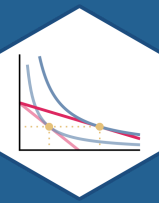
Digression: Measuring Change

Quantifying Changes I
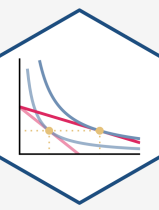
- Several ways we can talk about how a measure **changes** over time, from time $t_1 \rightarrow t_2$

- **Difference** (Δ)**:** the difference between the value at time t_1 and time t_2

$$\Delta t = t_2 - t_1$$

Quantifying Changes II

- Several ways we can talk about how a measure **changes** over time, from time $t_1 \rightarrow t_2$

- **Difference** (Δ): the difference between the value at time t_1 and time t_2
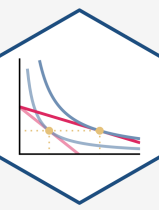
$$\Delta t = t_2 - t_1$$

- **Relative Difference**: the difference expressed **in terms of the original value**

$$\frac{\Delta t}{t_1} = \frac{t_2 - t_1}{t_1}$$

this becomes a proportion (a decimal)

Quantifying Changes III
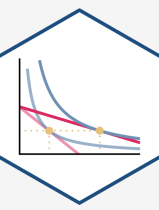
- **Percentage Change (Growth Rate)**: **relative** difference expressed as a **percentage**

$$\%\Delta = \frac{\Delta t}{t_1} \times 100\%$$

$$= \frac{t_2 - t_1}{t_1} \times 100\%$$

A Simple Example Growth Rate

Example: A country's GDP is $100bn in 2019, and $120bn in 2020. Calculate the country's GDP growth rate for 2020:

Elasticity, in General
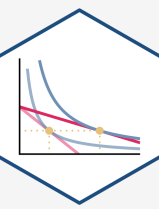
$$\epsilon_{y,x} = \frac{\%\Delta y}{\%\Delta x} = \frac{\frac{\Delta y}{y}}{\frac{\Delta x}{x}}$$
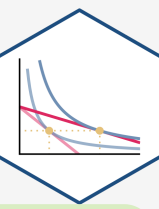
- An **elasticity** between any two variables y and x describes the **responsiveness** of a variable (y) to a change in another (x).
  - (relative change in y) over (relative change in x)

Elasticity, in General

$$\epsilon_{y,x} = \frac{\%\Delta y}{\%\Delta x} = \frac{\frac{\Delta y}{y}}{\frac{\Delta x}{x}}$$

- An **elasticity** between any two variables y and x describes the **responsiveness** of a variable (y) to a change in another (x).

    - (relative change in y) over (relative change in x)

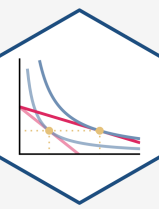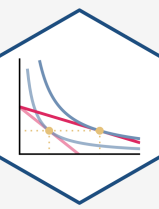- **Interpretation:** $\epsilon_{y,x} =$ **the *percentage change* in y from a *1%* change in x**

Elasticity, in General

$$\epsilon_{y,x} = \frac{\%\Delta y}{\%\Delta x} = \frac{\frac{\Delta y}{y}}{\frac{\Delta x}{x}}$$

- An **elasticity** between any two variables y and x describes the **responsiveness** of a variable (y) to a change in another (x).

  - (relative change in y) over (relative change in x)

- **Interpretation:** $\epsilon_{y,x}$ = the *percentage change* in y from a *1% change* in x

- **Unitless**: easy comparisons between **any 2 variables**

  - e.g. crime rates and police, GDP and gov't spending, inequality and corruption

Income Elasticity of Demand I
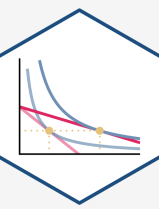
- The **income elasticity of demand** measures how much quantity demanded (q_D) changes in response to a change in income (m)

$$\epsilon_{q,m} = \frac{\%\Delta q_D}{\%\Delta m}$$

Income Elasticity of Demand I

- The **income elasticity of demand** measures how much quantity demanded (q_D) changes in response to a change in income (m)

$$\epsilon_{q,m} = \frac{\%\Delta q_D}{\%\Delta m}$$

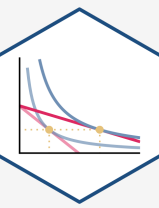- If $\epsilon_{q,m}$ is **negative**: an **inferior** good
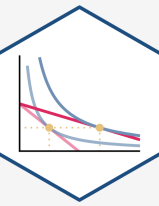
Income Elasticity of Demand I

- The **income elasticity of demand** measures how much quantity demanded (q_D) changes in response to a change in income (m)

$$\epsilon_{q,m} = \frac{\%\Delta q_D}{\%\Delta m}$$

- If $\epsilon_{q,m}$ is **negative**: an **inferior** good
- If $\epsilon_{q,m}$ is **positive**: a **normal** good
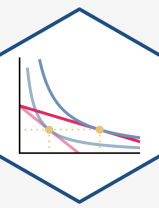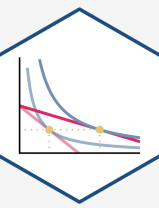
Income Elasticity of Demand I

- The **income elasticity of demand** measures how much quantity demanded (q_D) changes in response to a change in income (m)

$$\epsilon_{q,m} = \frac{\%\Delta q_D}{\%\Delta m}$$

- If $\epsilon_{q,m}$ is **negative**: an **inferior** good
- If $\epsilon_{q,m}$ is **positive**: a **normal** good
- Two subtypes of normal goods:
  - **Necessity**: $0 \leq \epsilon_{q,m} \leq 1$
    - ↑ quantity demanded as ↑↑ income (water, clothing)
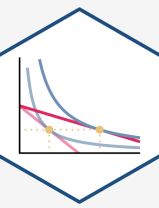
Income Elasticity of Demand I

- The **income elasticity of demand** measures how much quantity demanded (q_D) changes in response to a change in income (m)

$$\epsilon_{q,m} = \frac{\%\Delta q_D}{\%\Delta m}$$

- If $\epsilon_{q,m}$ is **negative**: an **inferior** good
- If $\epsilon_{q,m}$ is **positive**: a **normal** good
- Two subtypes of normal goods:
  - **Necessity**: $0 \leq \epsilon_{q,m} \leq 1$
    - ↑ quantity demanded as ↑↑ income (water, clothing)
  - **Luxury**: $\epsilon_{q,m} > 1$
    - ↑↑ quantity demanded as ↑ income (jewelry, vacations)
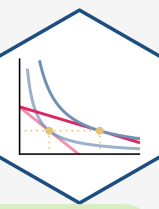
Income Elasticity of Demand II

- For now, we can **calculate** the income elasticity of demand simply by calculating the **relative changes**:

$$\frac{\%\Delta q}{\%\Delta m} = \frac{\left(\frac{\Delta q}{q_1}\right)}{\left(\frac{\Delta m}{m_1}\right)}$$

- We'll use some fancier methods when we talk about the only elasticity you've probably seen before: *price* elasticity of demand

Income Elasticity of Demand: Example

Example: You can spend your income on golf and pancakes. Green fees at a local golf course are $10 per round and pancake mix is $2 per box. When your income is $100, you buy 5 boxes of pancake mix and 9 rounds of golf. When your income increases to $120, you buy 10 boxes of pancake mix and 10 rounds of golf.

1. What type of good is golf (inferior, necessity, luxury)?

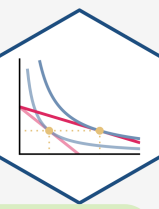2. What type of good are pancakes (inferior, necessity, or luxury)?
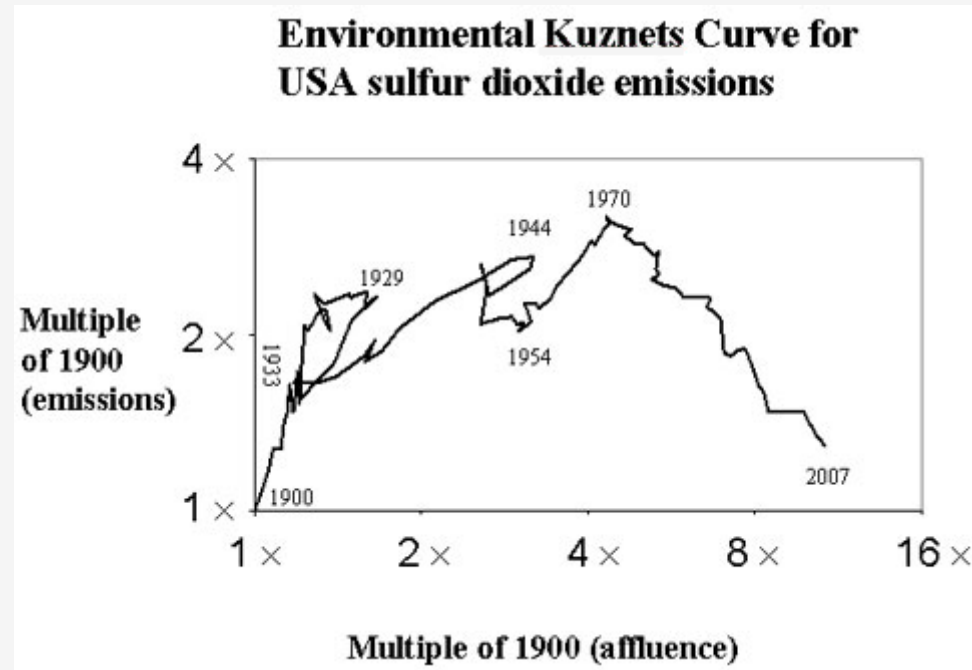
Income Effects: Example
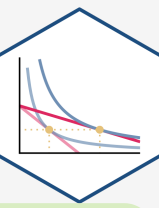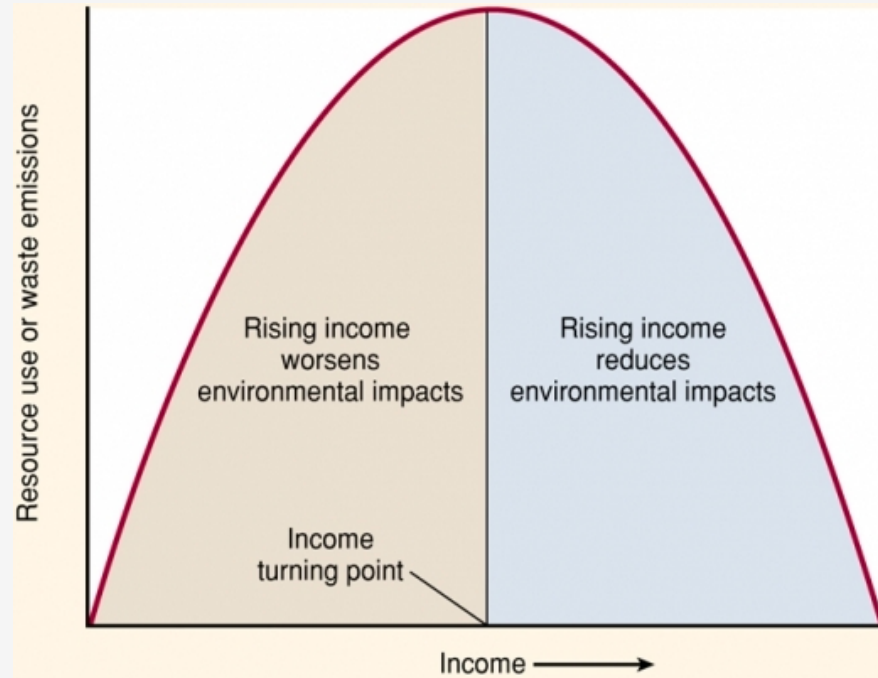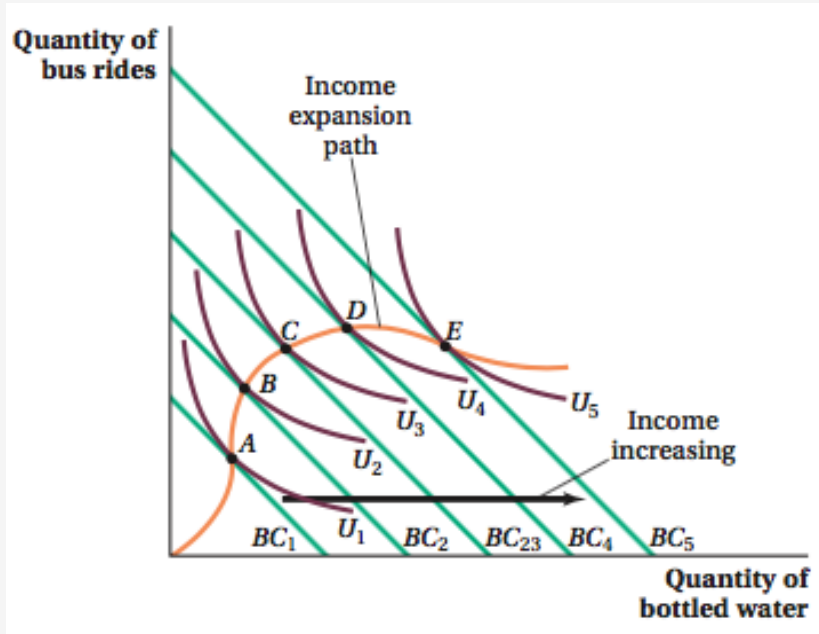
Example: Is the environment a normal good?

Income Effects: Example

Example: Is the environment a normal good?

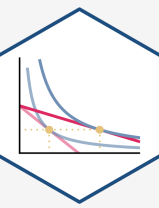Environmental Kuznets Curve for USA sulfur dioxide emissions
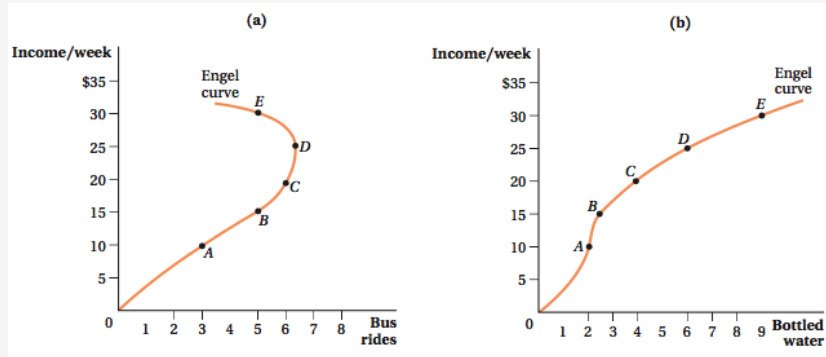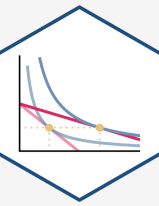
Income Expansion Path

Goolsbee, et. al (2011: 169)

- **Income expansion path** describes how consumption of each good changes when income increases
  - Traces a line between optimal consumption points as income increases (budget constraint shifts out)
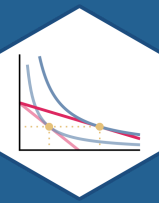
Engel Curves

Goolsbee, et. al (2011: 171)
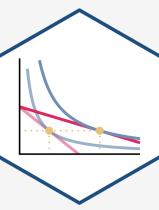
- **Engel curve** of each good is more helpful to visualize: shows how consumption of *one* good changes when income increases
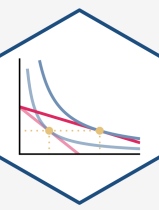
Cross-Price Effects

Cross-Price Effects

- **Cross-price effect**: change in optimal consumption of a good associated with a change in price of *another* good income, holding the good's *own* price (and income) constant

$$\frac{\Delta q_x}{\Delta p_y} >^? < 0$$

Cross-Price Elasticity of Demand I

- The **cross-price elasticity of demand** measures how much quantity demanded of one good (q_x) changes in response to a change in price of *another* good (p_y)

$$\epsilon_{q_x, p_y} = \frac{\% \Delta q_x}{\% \Delta p_y}$$
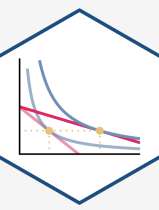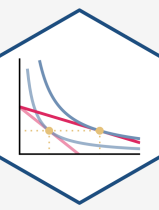
Cross-Price Elasticity of Demand I

- The **cross-price elasticity of demand** measures how much quantity demanded of one good (q_x) changes in response to a change in price of *another* good (p_y)

$$\epsilon_{q_x, p_y} = \frac{\% \Delta q_x}{\% \Delta p_y} = \frac{\frac{\Delta q_x}{q_x}}{\frac{\Delta p_y}{p_y}}$$
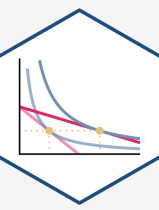
Cross-Price Elasticity of Demand II

$$\epsilon_{q_x, p_y} = \frac{\%\Delta q_x}{\%\Delta p_y}$$

- If ϵ_{q_x, p_y} is *positive*: goods x and y are **substitutes**

- An rise (fall) in price of y causes more (less) consumption of x

    - Consumption of x moves in **same direction** as price of y
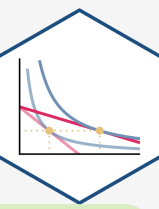
Cross-Price Elasticity of Demand III

$$\epsilon_{q_x, p_y} = \frac{\% \Delta q_x}{\% \Delta p_y}$$

- If ϵ_{q_x, p_y} is *negative*: goods x and y are **complements**

- Goods x and y consumed in a *bundle*, concern about overall price of *bundle*

- A rise (fall) in price of y causes less (more) consumption of x

    - Consumption of x moves in **opposite direction** as price of y

FREE

© StockFreeImages.com   443100

Cross-Price Elasticity: Example I

Example: You can travel into the city every week on Lyft rides and Uber rides. When Lyft is $20/ride, you ride 10 Uber rides. When Lyft raises prices to $25/ride, you ride 15 Uber rides.

1. What is the relationship between these two goods?

2. What is the cross-price elasticity?