1.4 — Preferences and Utility

ECON 306 • Microeconomic Analysis • Fall 2020

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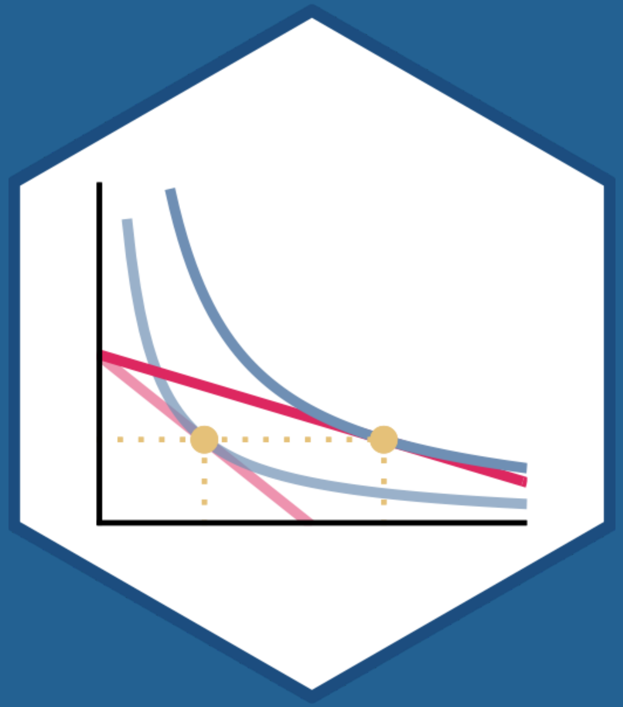

Outline

Consumer's Objectives

- What do consumers want? What do they ***maximize***?

- Avoid being normative & make as few assumptions as possible

- We'll assume people maximize **preferences**

  - WTF does that mean?

Preferences

Preferences I

- Which bundles of (x, y) are **preferred** over others?

Example:

$$a = \begin{pmatrix} 4 \\ 12 \end{pmatrix} \text{ or } b = \begin{pmatrix} 6 \\ 12 \end{pmatrix}$$

Preferences II

- We will allow **three possible answers**:

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1. $a \succ b$: Strictly prefer a over b

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3. $a \sim b$: Indifferent between a and b

Preferences II

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3. $a \sim b$: Indifferent between a and b

- ***Preferences*** **are a list of all such comparisons between all bundles**

See appendix in today's class page for more.

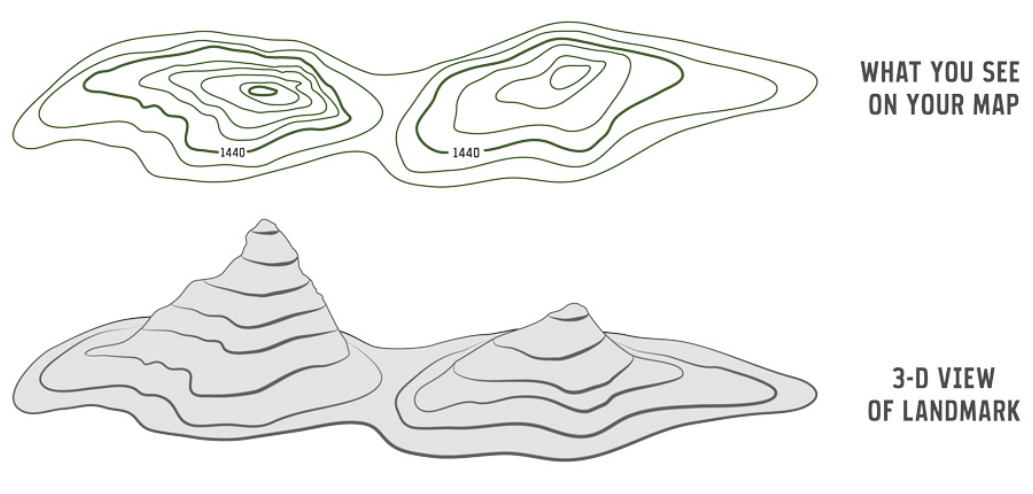
Indifference Curves

Mapping Preferences Graphically I

- For each bundle, we now have 3 pieces of information:
    - amount of x
    - amount of y
    - preference compared to other bundles
- How to represent this information graphically?

Mapping Preferences Graphically II

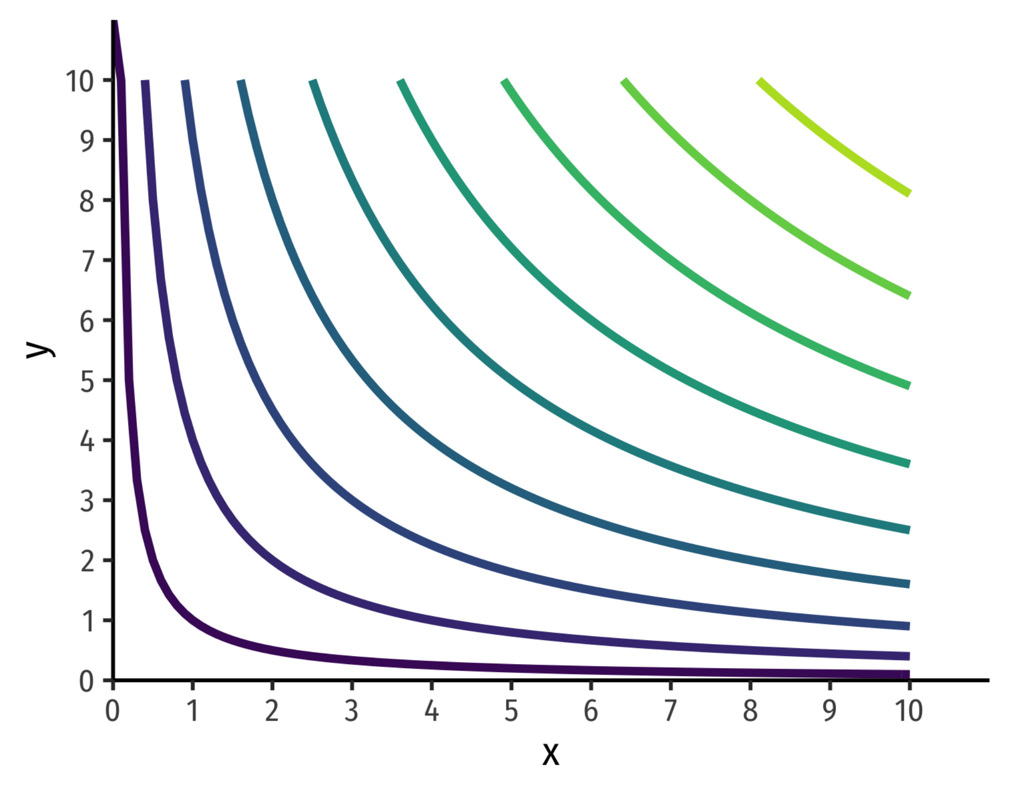
- Cartographers have the answer for us
- On a map, **contour lines** link areas of **equal height**
- We will use "indifference curves" to link bundles of **equal preference**



Mapping Preferences Graphically III

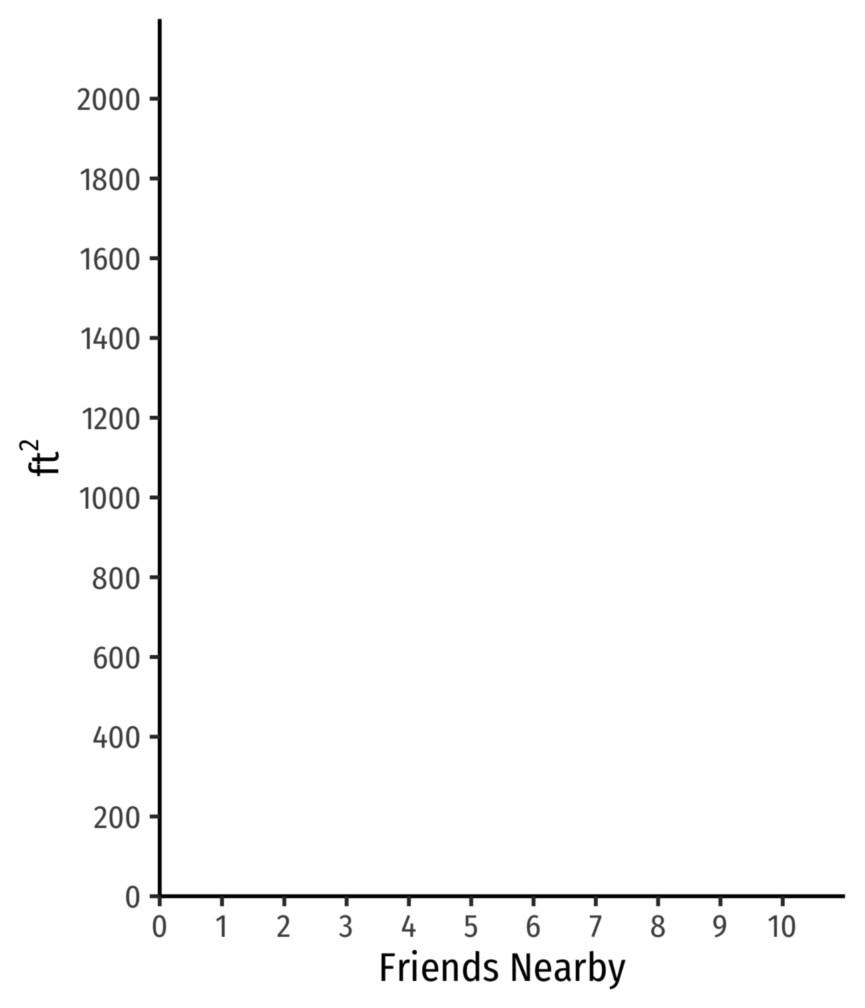
3-D "Mount Utility"

2-D Indifference Curve Contours



Indifference Curves: Example

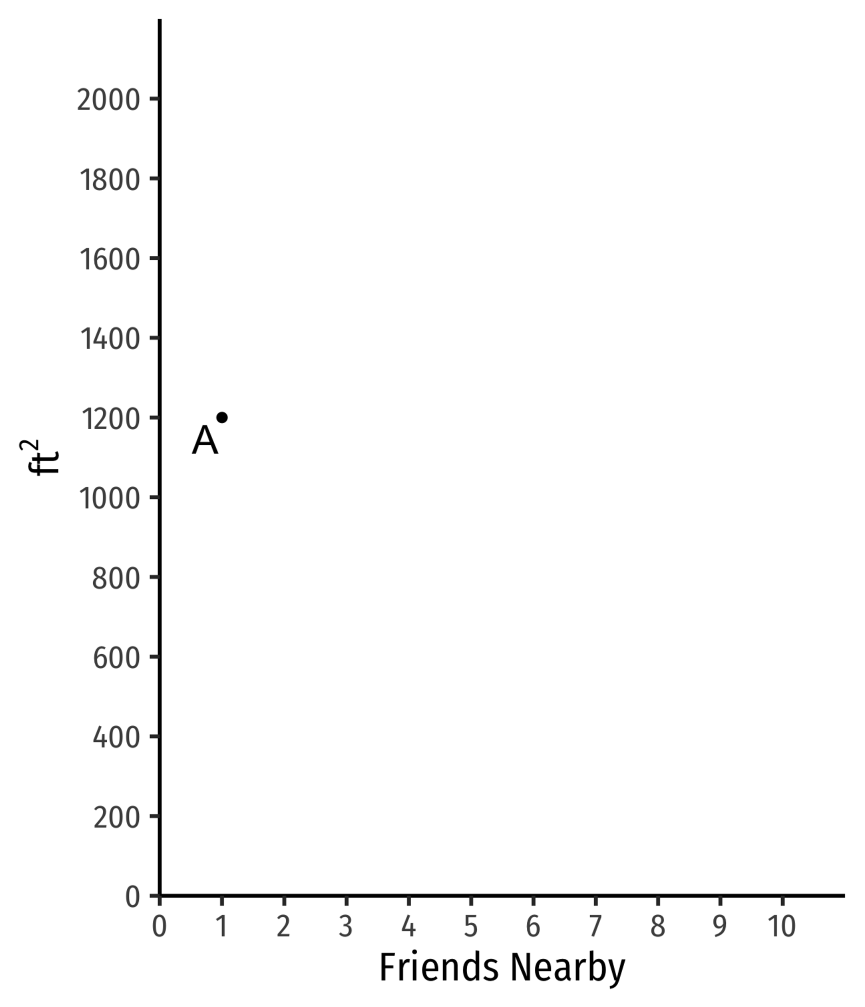
Example: Suppose you are hunting for an apartment. You value *both* the size of the apartment and the number of friends that live nearby.



Indifference Curves: Example

Example: Suppose you are hunting for an apartment. You value *both* the size of the apartment and the number of friends that live nearby.

- Apt. *A* has 1 friend nearby and is 1,200 ft^2



Indifference Curves: Example

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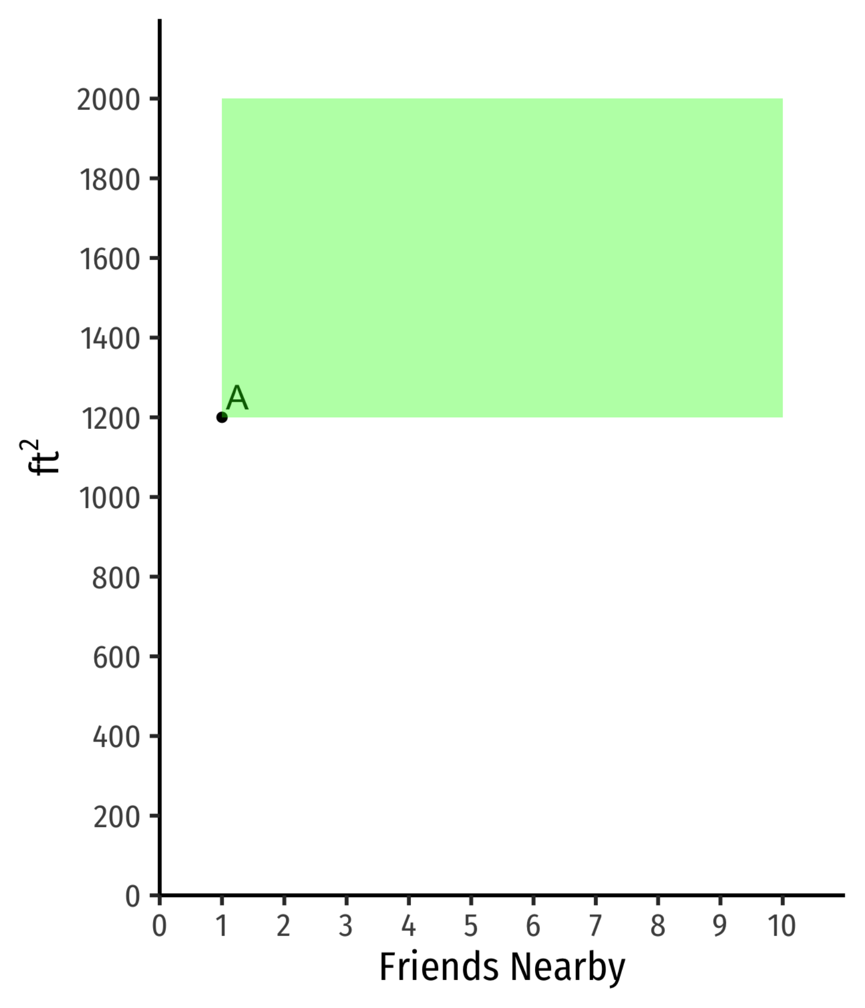
  - Apartments that are larger and/or have more friends $\succ A$



Indifference Curves: Example

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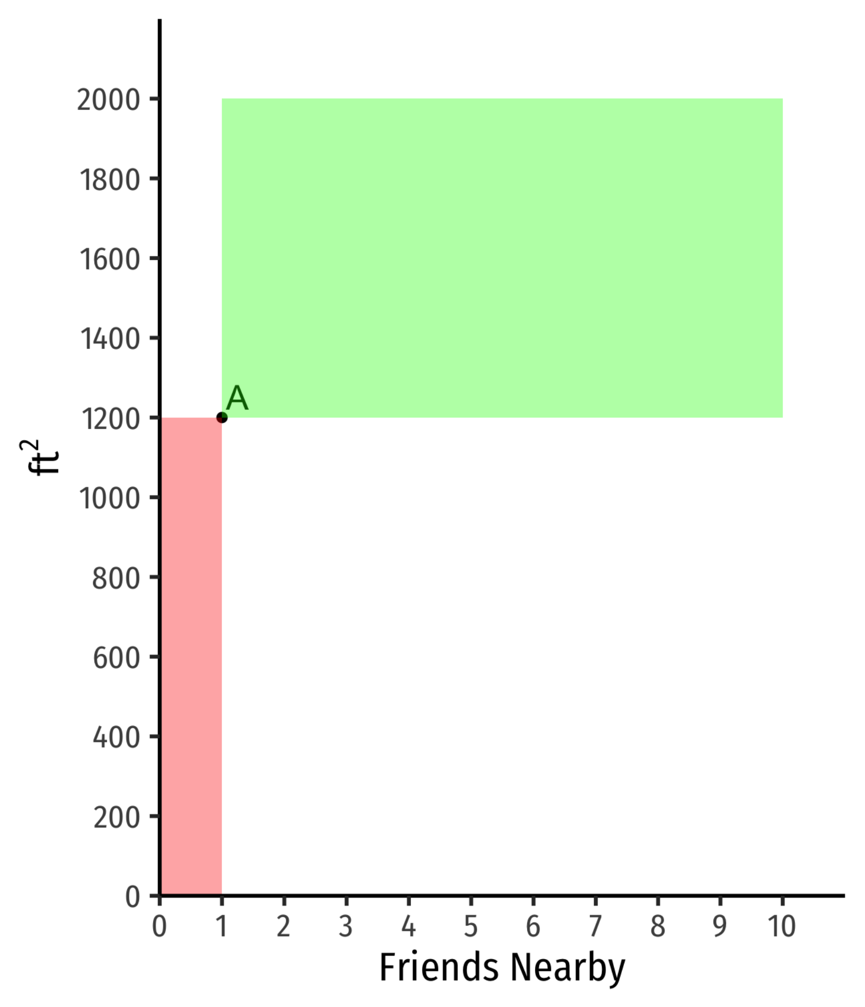
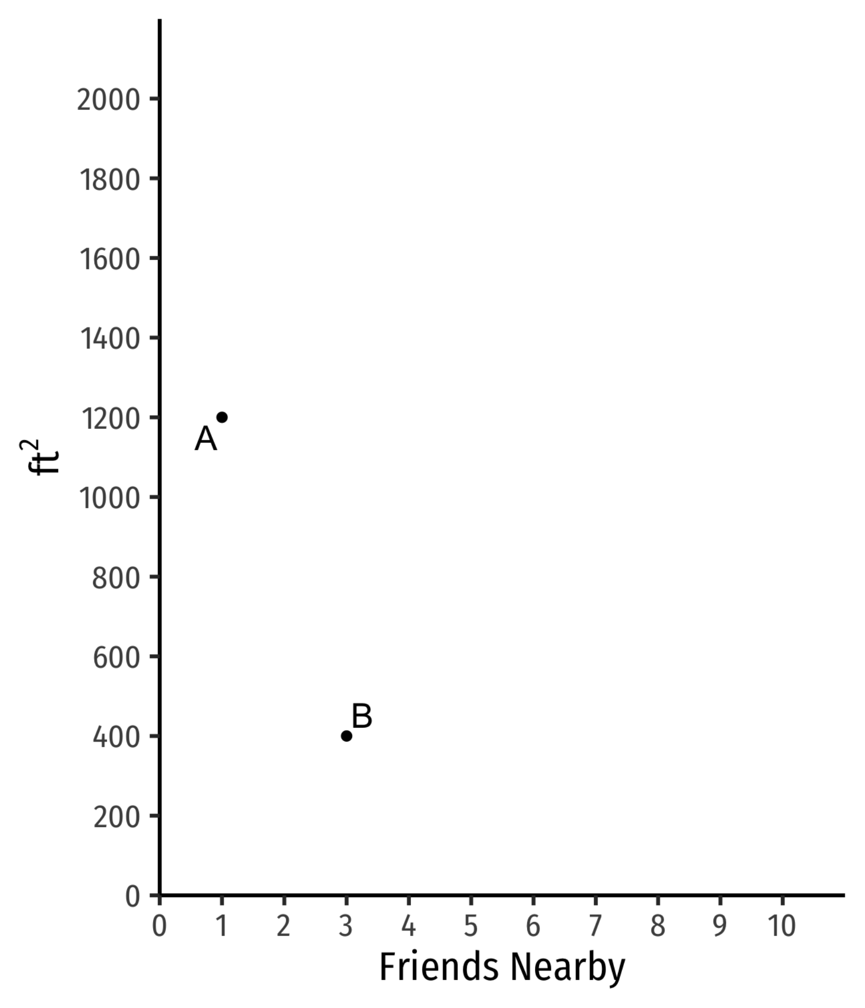
- Apt. *A* has 1 friend nearby and is 1,200 ft^2
  - Apartments that are larger and/or have more friends $\succ A$
  - Apartments that are smaller and/or have fewer friends $\prec A$



Indifference Curves: Example

Example:

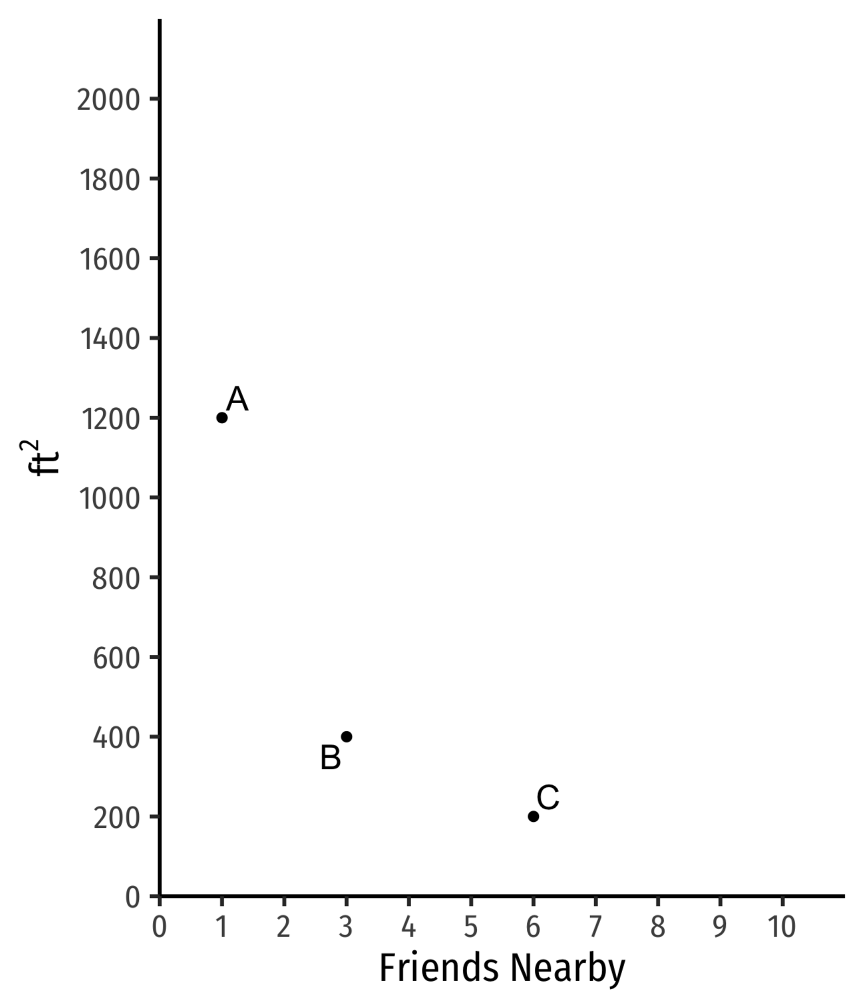
- Apt. *A* has 1 friend nearby and is 1,200 ft^2
- Apt. *B* has *more* friends but *less* ft^2



Indifference Curves: Example

Example:

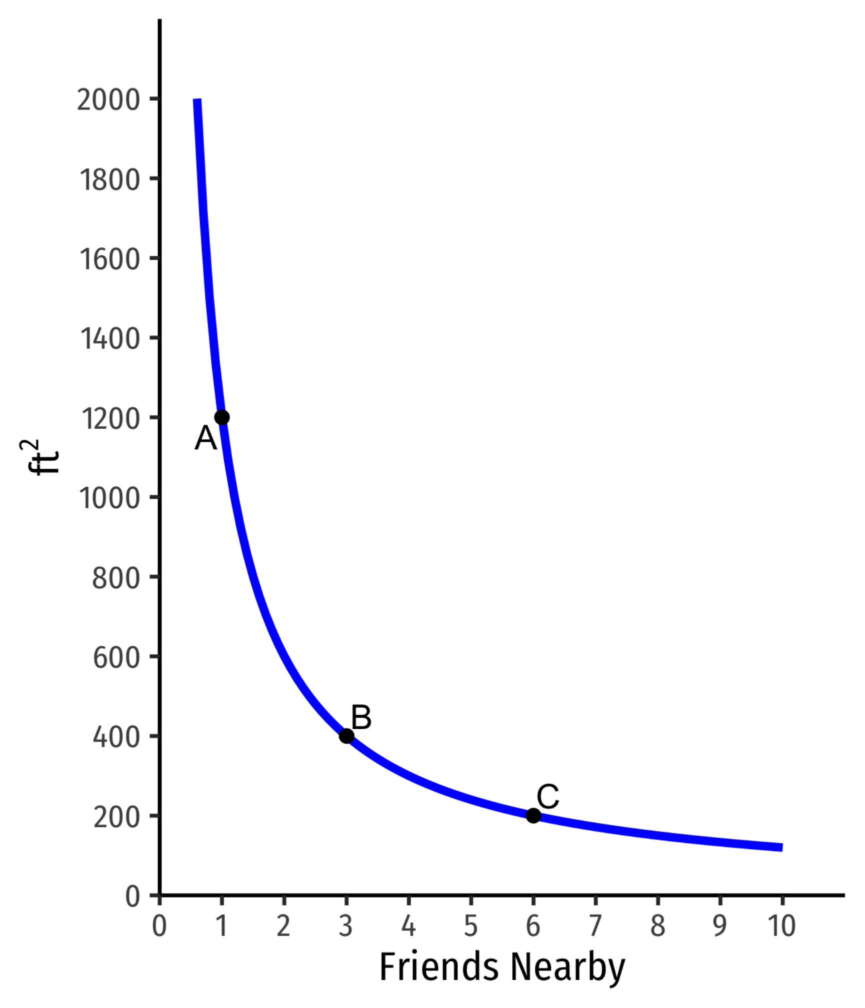
- Apt. *A* has 1 friend nearby and is 1,200 ft^2
- Apt. *B* has *more* friends but *less* ft^2
- Apt. *C* has *still more* friends but *less* ft^2



Indifference Curves: Example

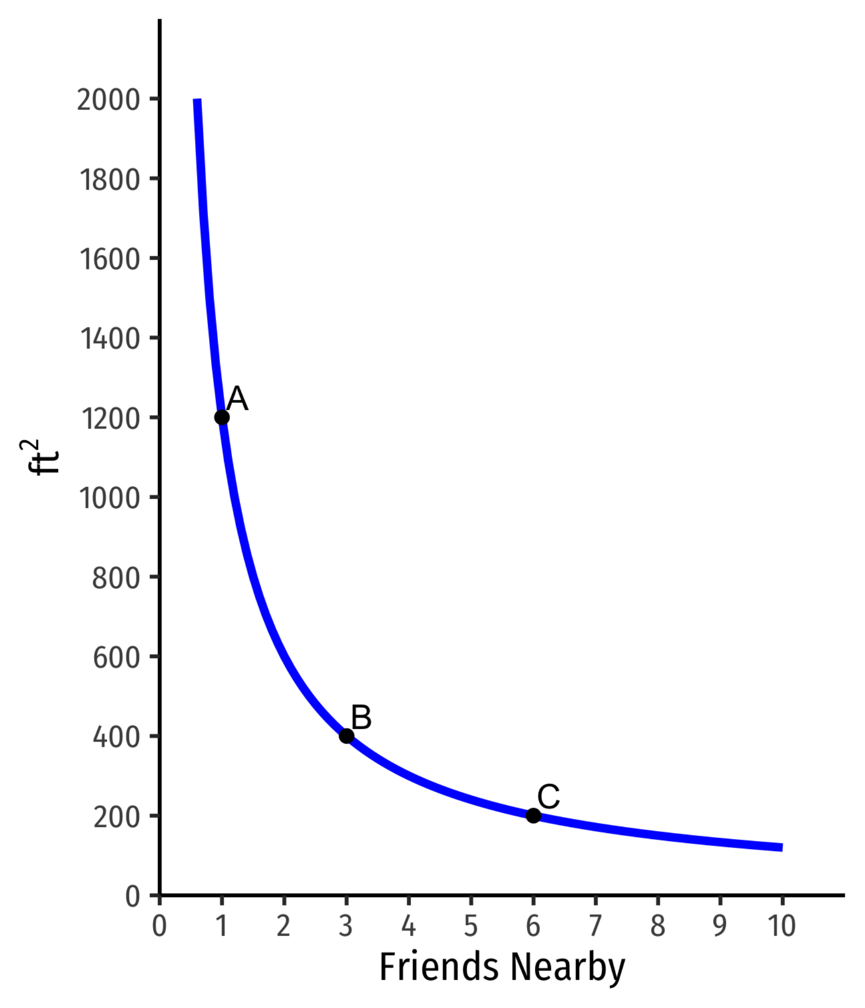
Example:

- Apt. *A* has 1 friend nearby and is 1,200 ft^2
- Apt. *B* has *more* friends but *less* ft^2
- Apt. *C* has *still more* friends but *less* ft^2
- If $A \sim B \sim C$, on same **indifference curve**



Indifference Curves: Example

- **Indifferent** between all apartments on the **same** curve

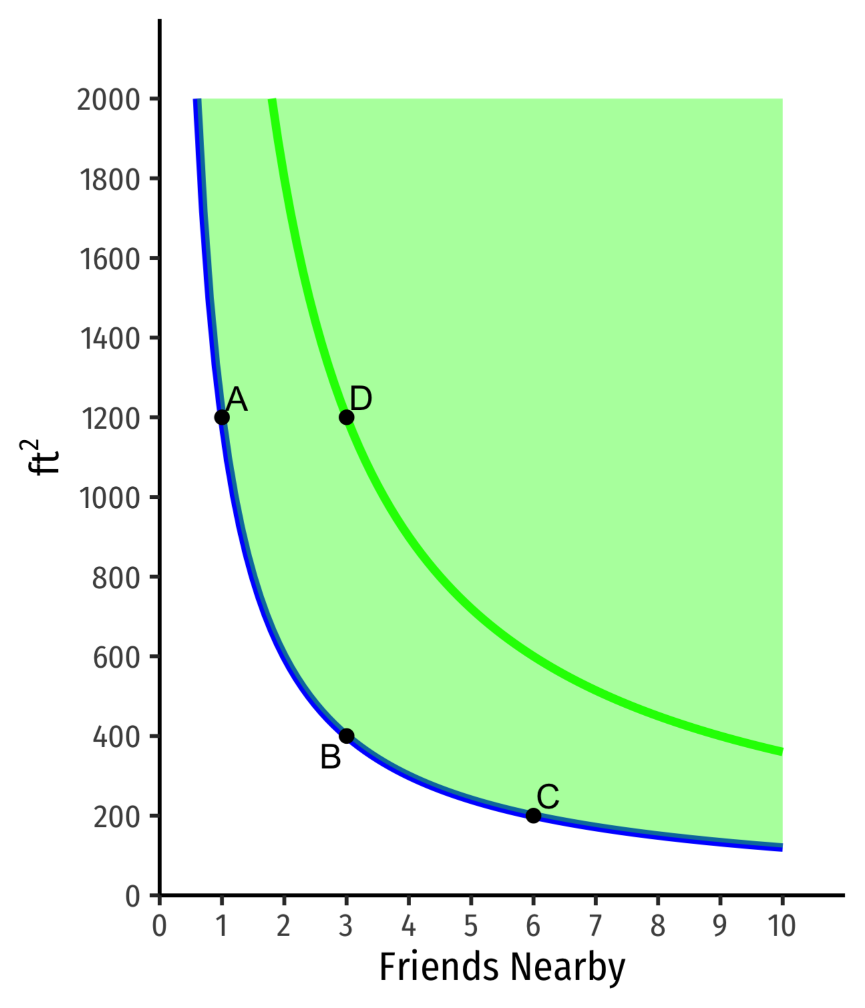


Indifference Curves: Example

- **Indifferent** between all apartments on the **same** curve

- Apts **above** curve are **preferred over** apts on curve

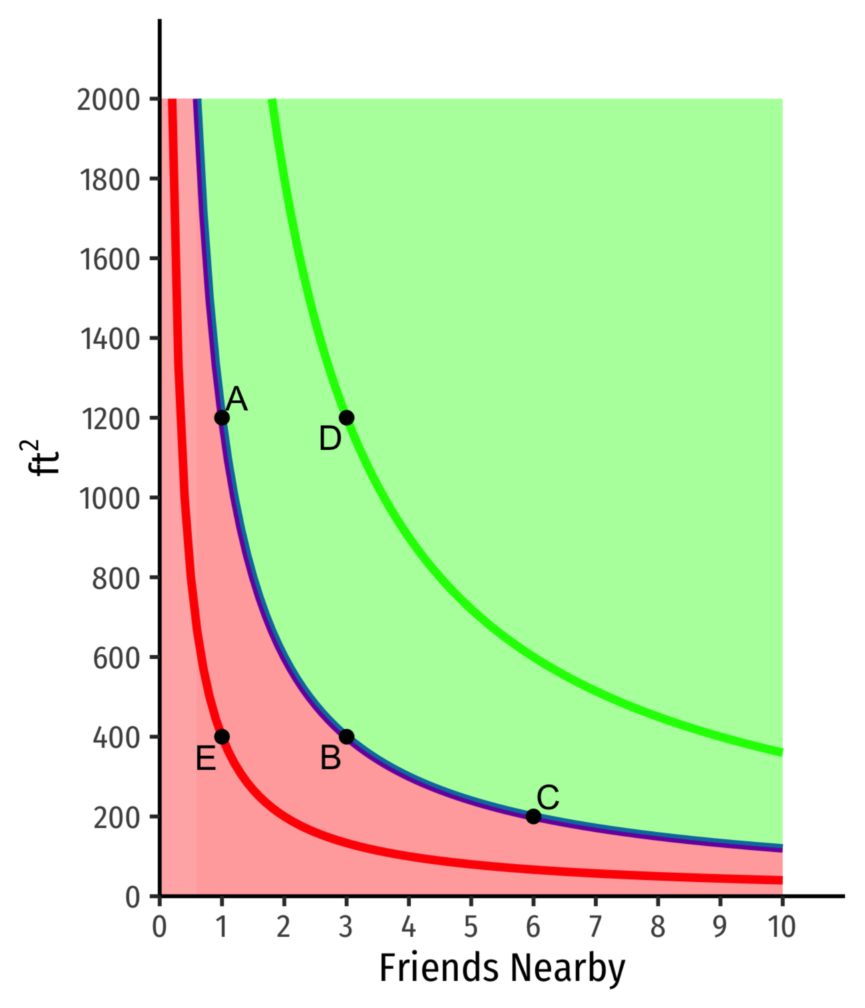
    - $D \succ A \sim B \sim C$
    - On a **higher curve**



Indifference Curves: Example

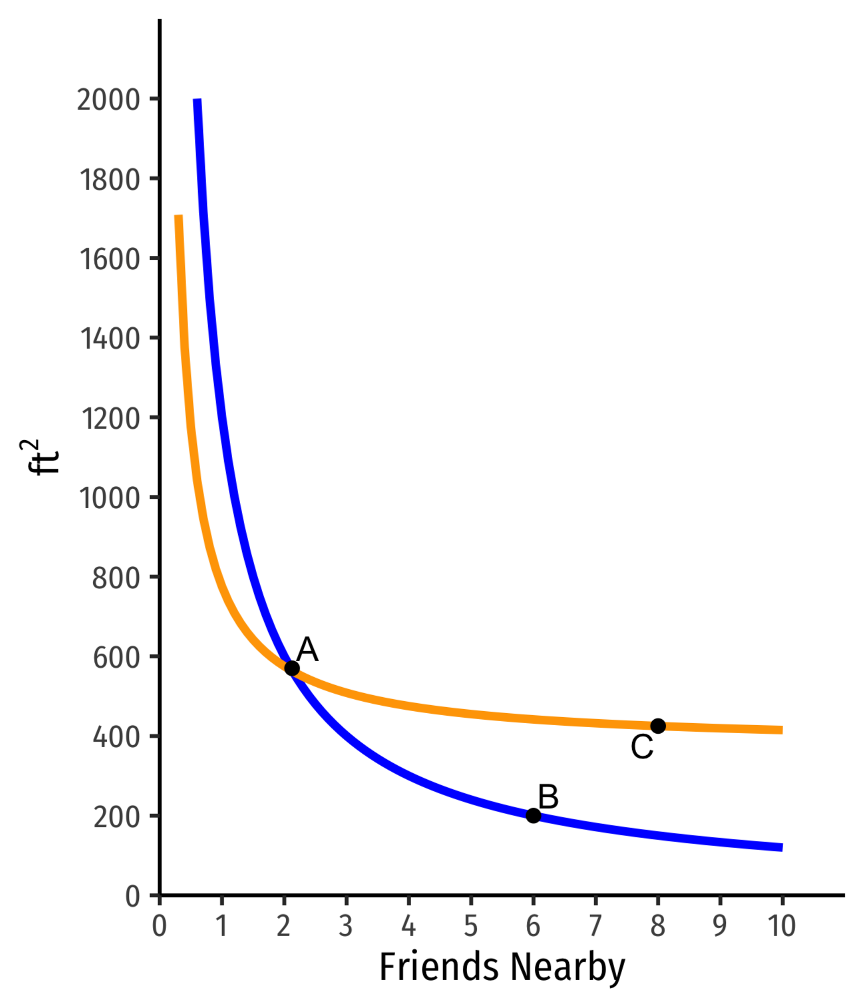
- **Indifferent** between all apartments on the **same** curve

- Apts **above** curve are **preferred over** apts on curve
  - $D \succ A \sim B \sim C$
  - On a **higher curve**
- Apts **below** curve are **less preferred** than apts on curve
  - $E \prec A \sim B \sim C$
  - On a **lower curve**



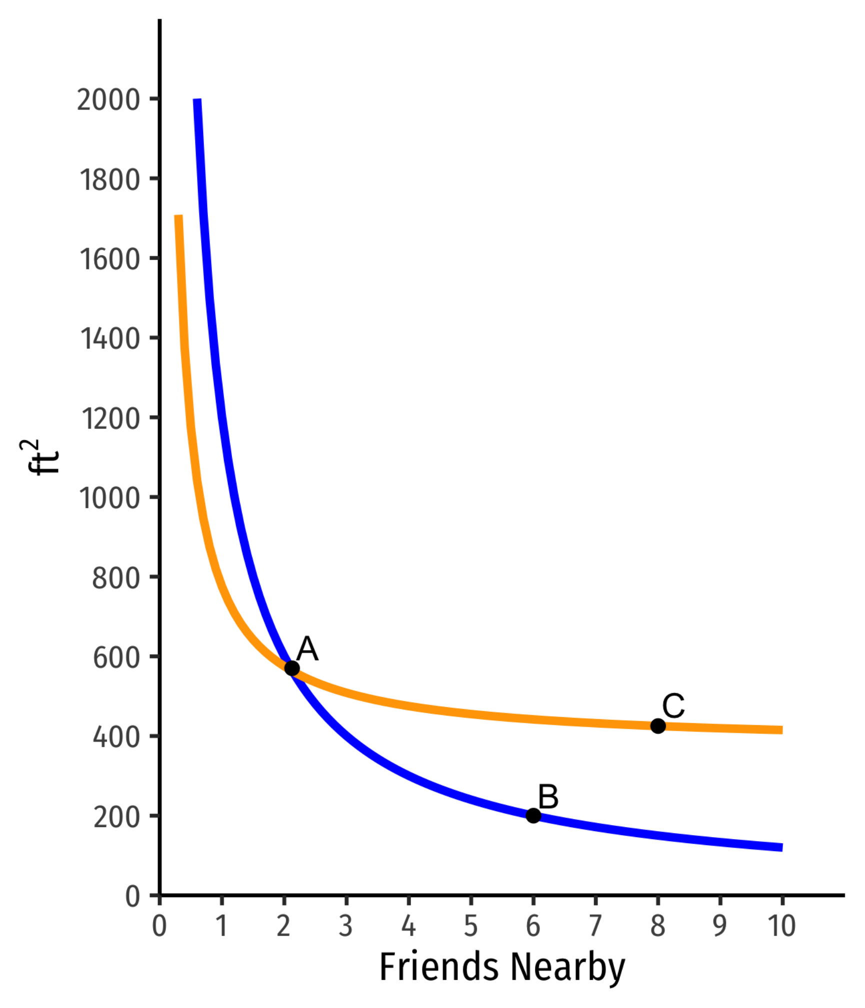
Curves Never Cross!

- **Indifference curves can never cross**: preferences are transitive
  - If I prefer $A \succ B$, and $B \succ C$, I must prefer $A \succ C$



Curves Never Cross!

- **Indifference curves can never cross**: preferences are transitive
  - If I prefer $A \succ B$, and $B \succ C$, I must prefer $A \succ C$
- Suppose two curves crossed:
  - $A \sim B$
  - $B \sim C$
  - But $C \succ B$!
  - Preferences are not transitive!



Marginal Rate of Substitution

Marginal Rate of Substitution I

- If I take away one friend nearby, how many more ft^2 would you need to keep you **indifferent**?

Marginal Rate of Substitution I

- If I take away one friend nearby, how many more ft^2 would you need to keep you **indifferent**?
- **Marginal Rate of Substitution (MRS)**: rate at which you trade off one good for the other and remain *indifferent*
- Think of this as your **opportunity cost**: # of units of y you need to give up to acquire 1 more x

MRS vs. Budget Constraint Slope

- Budget constraint (slope) measured the **market's** tradeoff between x and y based on market prices

- **MRS** measures your **personal** evaluation of x vs. y based on your preferences

- **Foreshadowing**: what if they are *different*? Are you truly maximizing your preferences?

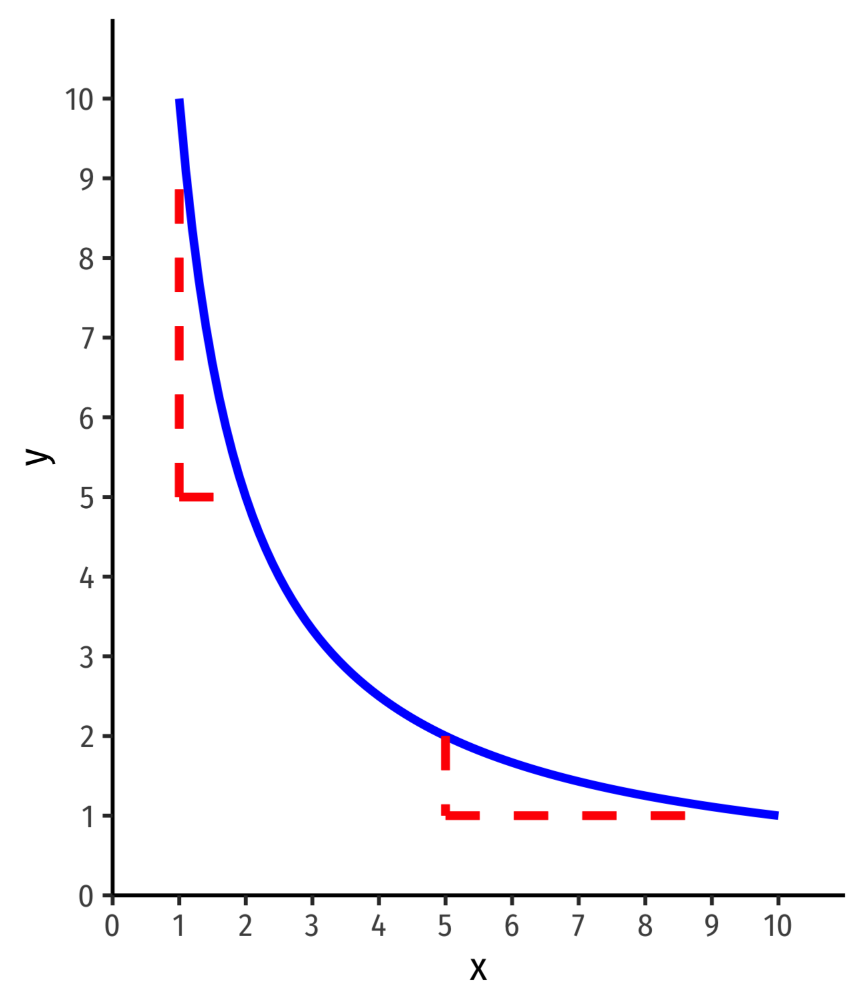
Marginal Rate of Substitution II

- MRS is the slope of the indifference curve

$$MRS_{x,y} = -\frac{\Delta y}{\Delta x} = \frac{rise}{run}$$

- Amount of y given up for 1 more x

- Note: slope (MRS) changes along the curve!



Utility

So Where are the Numbers?

- Long ago (1890s), utility considered a real, measurable, cardinal scale†
- Utility thought to be lurking in people's brains
  - Could be understood from first principles: calories, water, warmth, etc
- Obvious problems

† "Neuroeconomics" & cognitive scientists are re-attempting a scientific approach to measure utility

Utility Functions?

- 20th century innovation: **preferences** as the objects of maximization

- We can plausibly *measure* preferences via implications of peoples' actions!

- **Principle of Revealed Preference**: if x and y are both feasible, and if x is chosen over y, then the person must (weakly) prefer $x \succeq y$

- Flawless? Of course not. But extremely useful!

Utility Functions! I

- So how can we build a function to "maximize preferences"?

- Construct a **utility function** $u(\cdot)$[†] that *represents* preference relations $(\succ, \prec, \sim)$

- Assign utility numbers to bundles, such that, for any bundles a and b:

$$a \succ b \iff u(a) > u(b)$$

[†] The $\cdot$ is a placeholder for whatever goods we are considering (e.g. x, y, burritos, lattes, etc)

Utility Functions! II

- We can model "**as if**" the consumer is maximizing utility/preferences by **maximizing the utility function**:

- **"Maximizing preferences"**: choosing a such that $a \succ b$ for all available b

- **"Maximizing utility"**: choosing a such that $u(a) > u(b)$ for all available b

- Identical if they contain the same information

Utility Functions, Pural I

- Imagine three alternative bundles of (x, y):

$$\begin{aligned} a &= (1, 2) \\ b &= (2, 2) \\ c &= (4, 3) \end{aligned}$$

- Create a utility function $u(\cdot)$ that assigns each bundle a utility level of

$u(\cdot)$
$u(a) = 1$
$u(b) = 2$
$u(c) = 3$

Utility Functions, Pural I

- Imagine three alternative bundles of (x, y):

$$
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\end{aligned}
$$

- Create a utility function $u(\cdot)$ that assigns each bundle a utility level of

$u(\cdot)$
$u(a) = 1$
$u(b) = 2$
$u(c) = 3$

- Does it mean that bundle c is 3 times the utility of a?

Utility Functions, Pural II

- Imagine three alternative bundles of (x, y):

$$\begin{aligned} a &= (1, 2) \\ b &= (2, 2) \\ c &= (4, 3) \end{aligned}$$

- Now consider $u(\cdot)$ and a *second* utility function $v(\cdot)$:

$u(\cdot)$	$v(\cdot)$
$u(a) = 1$	$v(a) = 3$
$u(b) = 2$	$v(b) = 5$
$u(c) = 3$	$v(c) = 7$

Utility Functions, Pural III

- Utility numbers have an **ordinal** meaning only, **not cardinal**
    - **Only the ordering** $c \succ b \succ a$ **matters!**

- Both are valid:[†]
    - $u(c) > u(b) > u(a)$
    - $v(c) > v(b) > v(a)$

[†] See the Mathematical Appendix in Today's Class Page for why.

Utility Functions and Indifference Curves I

- Two tools to represent preferences: **indifference curves** and **utility functions**

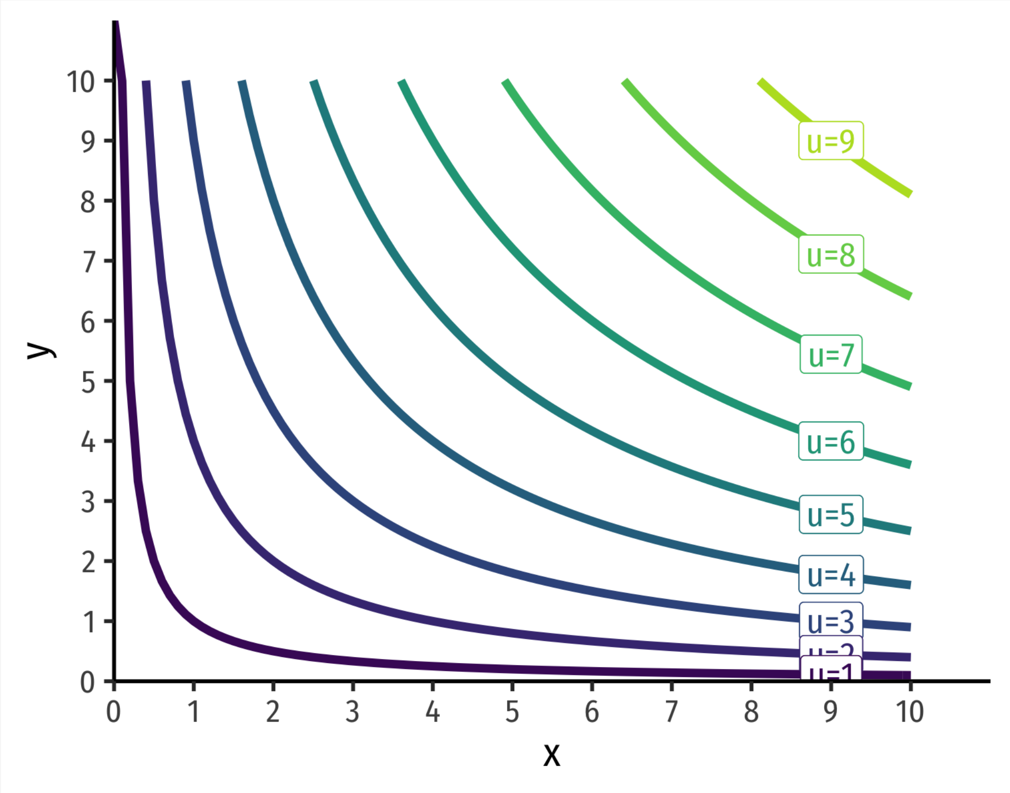
- Indifference curve: all **equally preferred** bundles $\iff$ **same utility level**

- Each indifference curve represents one level (or contour) of utility surface (function)

Utility Functions and Indifference Curves II

3-D Utility Function: $u(x, y) = \sqrt{xy}$

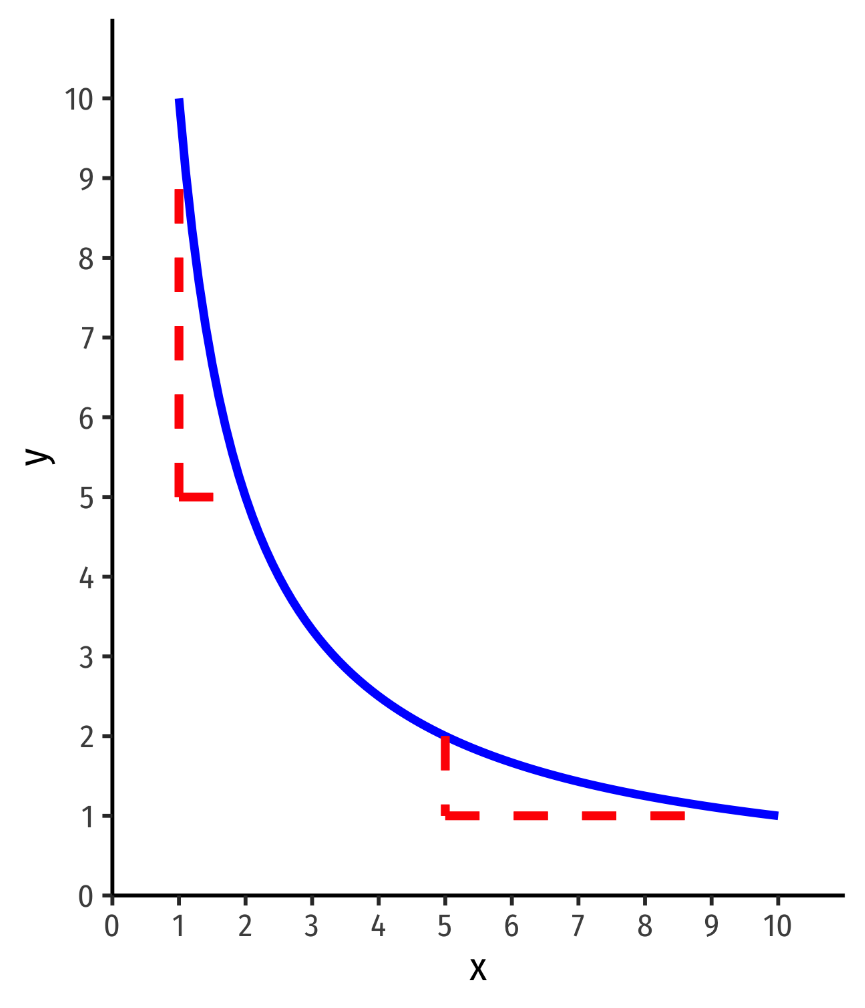
2-D Indifference Curve Contours: $y = \dfrac{u^2}{x}$



Marginal Utility

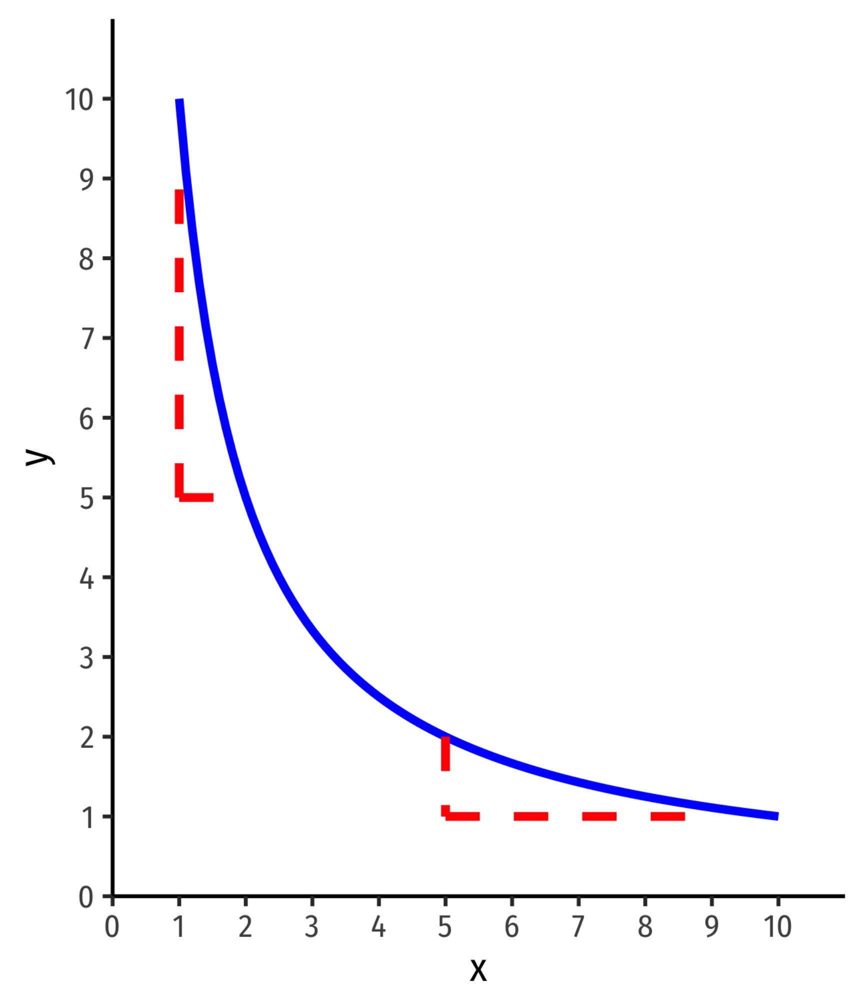
MRS and Marginal Utility I

- Recall: **marginal rate of substitution** $MRS_{x,y}$ is slope of the indifference curve
  - Amount of y given up for 1 more x
- How to calculate MRS?
  - Recall it changes (not a straight line)!
  - We can calculate it using something from the **utility function**



MRS and Marginal Utility II

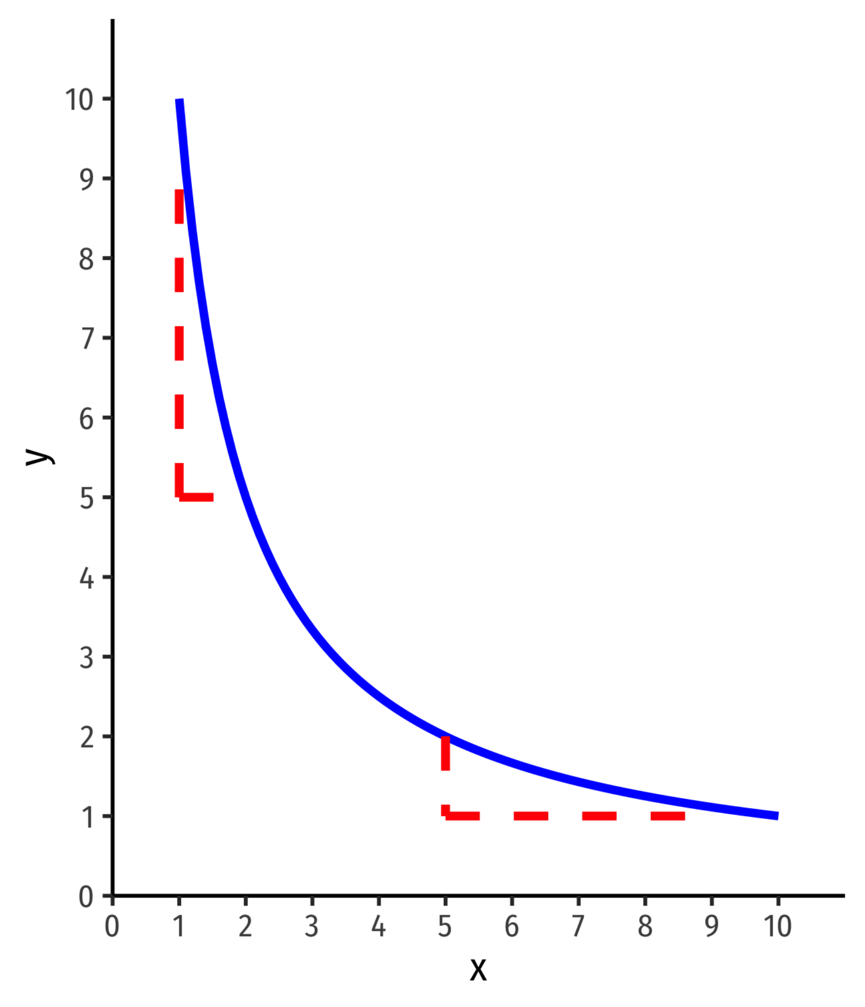
- **Marginal utility**: change in utility from a marginal increase in consumption



MRS and Marginal Utility II

- **Marginal utility**: change in utility from a marginal increase in consumption

Marginal utility of x: $MU_x = \dfrac{\Delta u(x,y)}{\Delta x}$

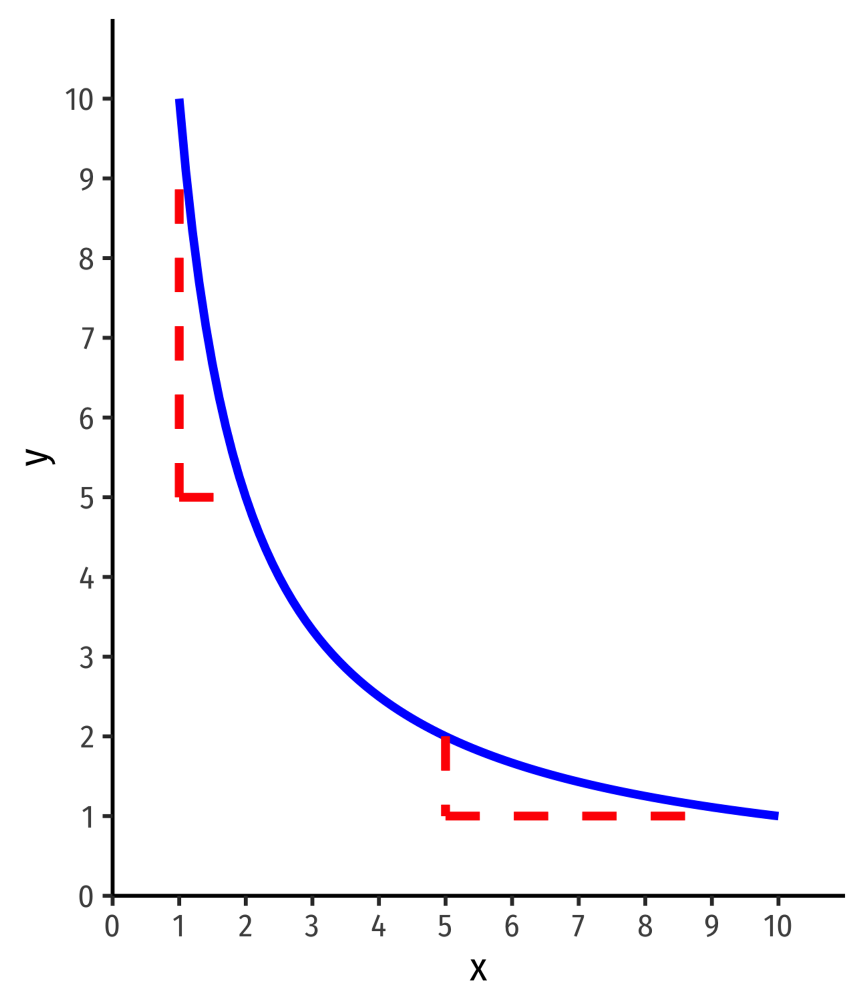


MRS and Marginal Utility II

- **Marginal utility**: change in utility from a marginal increase in consumption

Marginal utility of x: $MU_x = \dfrac{\Delta u(x,y)}{\Delta x}$

Marginal utility of y: $MU_y = \dfrac{\Delta u(x,y)}{\Delta y}$

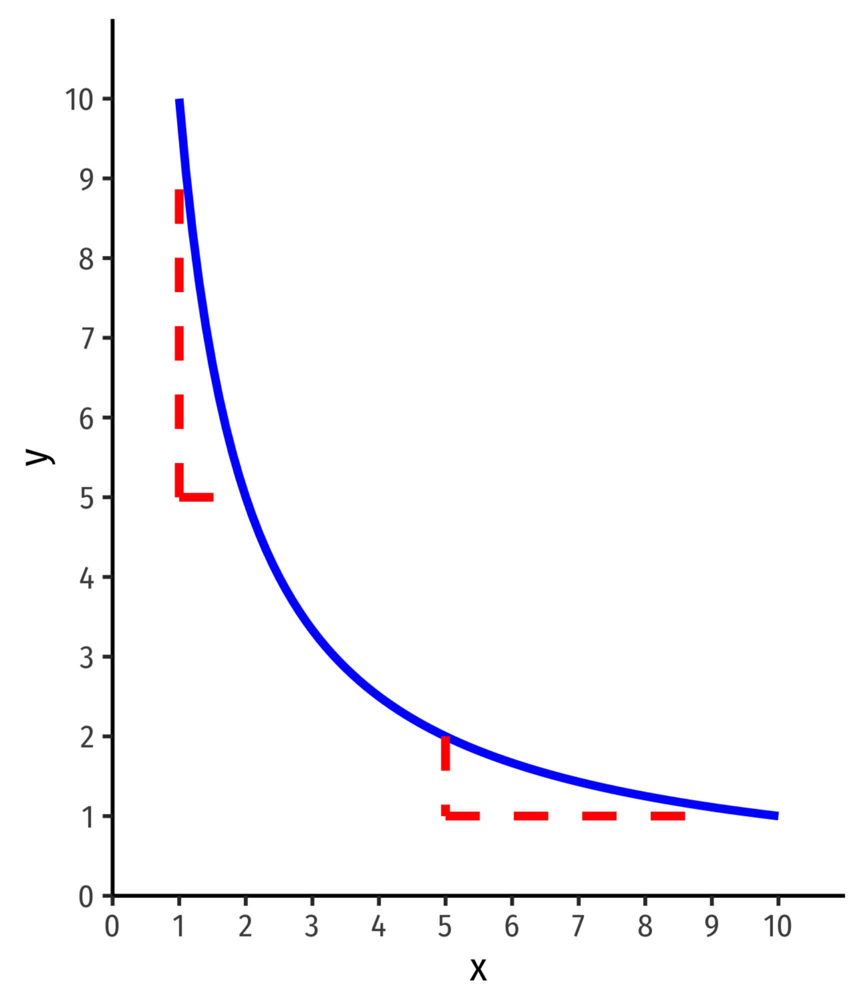


MRS and Marginal Utility II

- **Marginal utility**: change in utility from a marginal increase in consumption

- Math (calculus): "*marginal*" means "*derivative with respect to*"

  - I will always derive marginal utility functions for you



MRS and Marginal Utility: Example

Example: For an example utility function

$$u(x, y) = x^2 + y^3$$

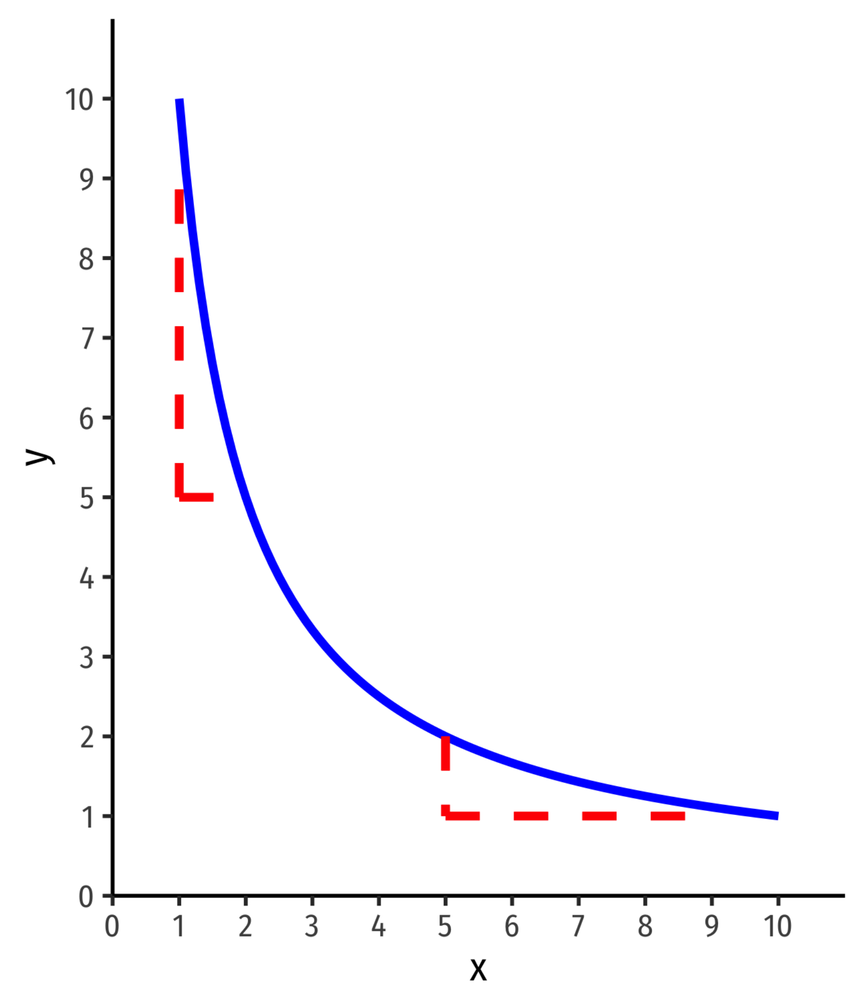
- Marginal utility of x: $MU_x = 2x$
- Marginal utlity of y: $MU_y = 3y^2$

MRS Equation and Marginal Utility

- Relationship between MU and MRS:

$$\underbrace{\frac{\Delta y}{\Delta x}}_{MRS} = -\frac{MU_x}{MU_y}$$

- See proof in [today's class notes]



MRS and Preferences

MRS and Preferences: Goods, Bads, Neutrals

- More precise ways to classify objects:
- A **good** enters utility function positively
  - $\uparrow$ good $\implies$ $\uparrow$ utility
  - Willing to pay (give up other goods) to *acquire more* (monotonic)

MRS and Preferences: Goods, Bads, Neutrals

- More precise ways to classify objects:
- A **good** enters utility function positively
  - $\uparrow$ good $\implies$ $\uparrow$ utility
  - Willing to pay (give up other goods) to *acquire more* (monotonic)
- A **bad** enters utility function negatively
  - $\uparrow$ good $\implies$ $\downarrow$ utility
  - Willing to pay (give up other goods) to *get rid of*

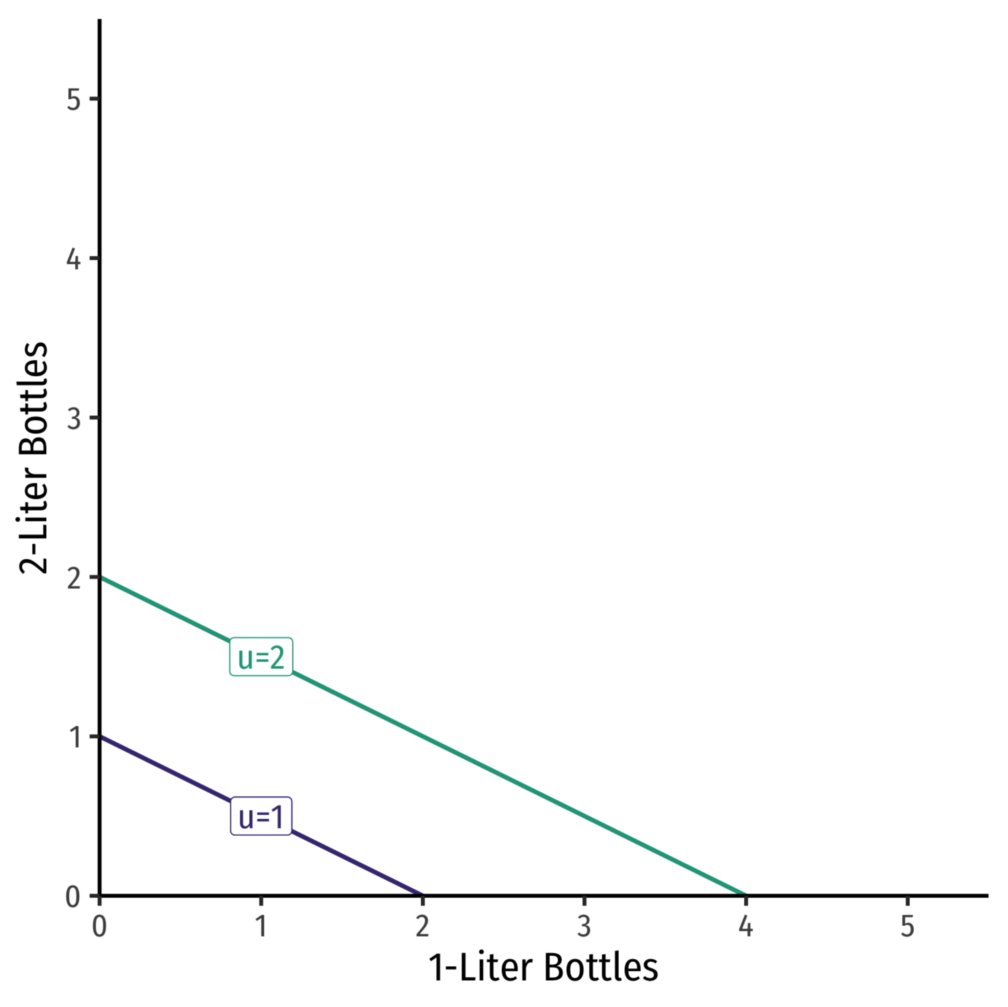
MRS and Preferences: Goods, Bads, Neutrals

- More precise ways to classify objects:
- A **neutral** does not enter utility function at all
  - ↑, ↓ has no effect on utility

MRS and Preferences: Substitutes

Example: Consider 1-Liter bottles of coke and 2-Liter bottles of coke

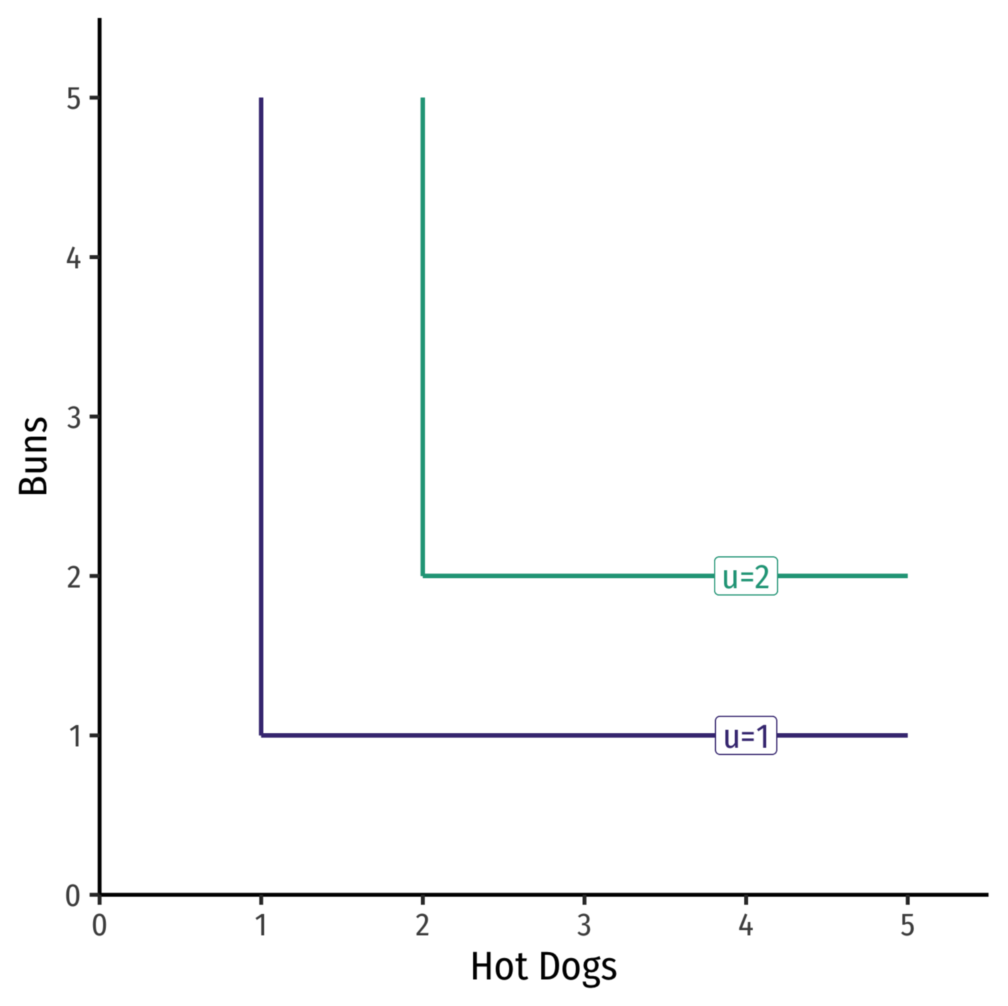
- Always willing to substitute between Two 1-L bottles for One 2-L bottle
- **Perfect substitutes**: goods that can be substituted at same fixed rate and yield same utility
- $MRS_{1L,2L} = -0.5$ (a constant!)



MRS and Preferences: Complements

Example: Consider hot dogs and hot dog buns

- Always consume together in fixed proportions (in this case, 1 for 1)
- **Perfect complements**: goods that can be consumed together in same fixed proportion and yield same utility
- $MRS_{H,B} = ?$

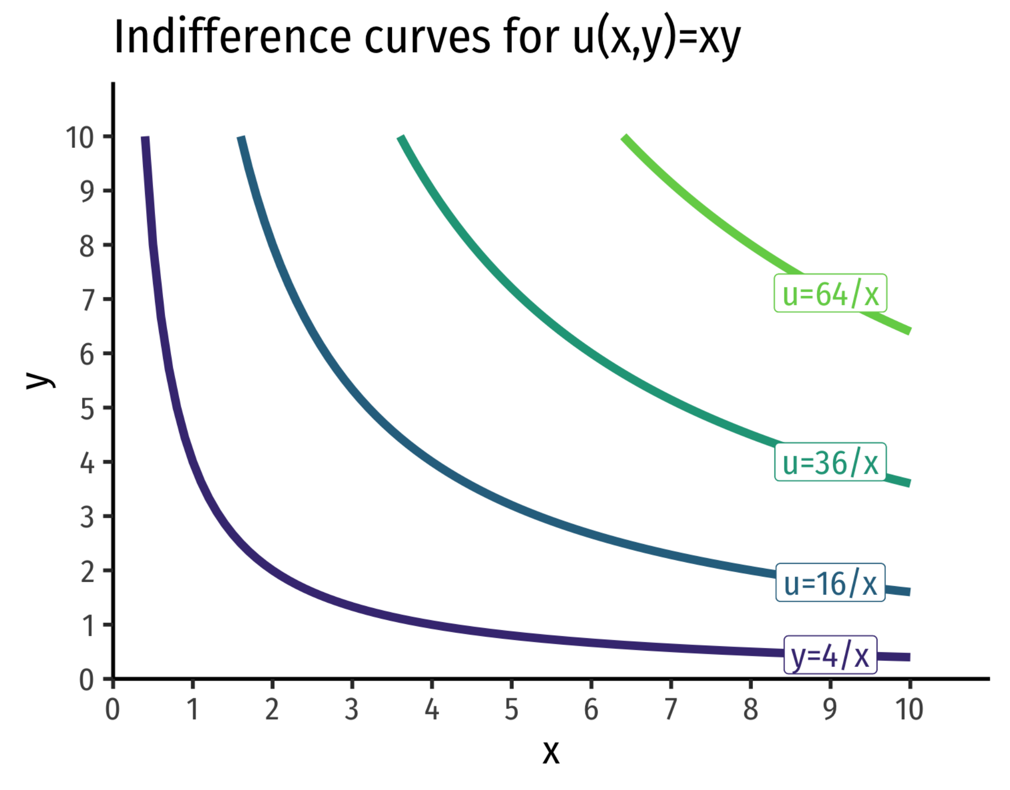


Cobb-Douglas Utility Functions

- A very common functional form in economics is **Cobb-Douglas**

$$u(x, y) = x^a y^b$$

- Where $a, b > 0$ (and very often $a + b = 1$)
- Extremely useful, you will see it often!
  - Strictly convex and monotonic indifference curves
  - Other nice properties (we'll see later)
  - See the appendix in [today's class page]()



Practice

Example: Suppose you can consume apples (a) and broccoli (b), and earn utility according to:

$$u(a,b) = 2ab$$

Where your marginal utilities are:

$$\begin{aligned} MU_a &= 2b \\ MU_b &= 2a \end{aligned}$$

1. Put a on the horizontal axis and b on the vertical axis. Write an equation for $MRS_{a,b}$.

2. Would bundles of $(1,4)$ and $(2,2)$ be on the same indifference curve?