1.5 — Solving the Consumer's Problem

ECON 306 • Microeconomic Analysis • Fall 2020

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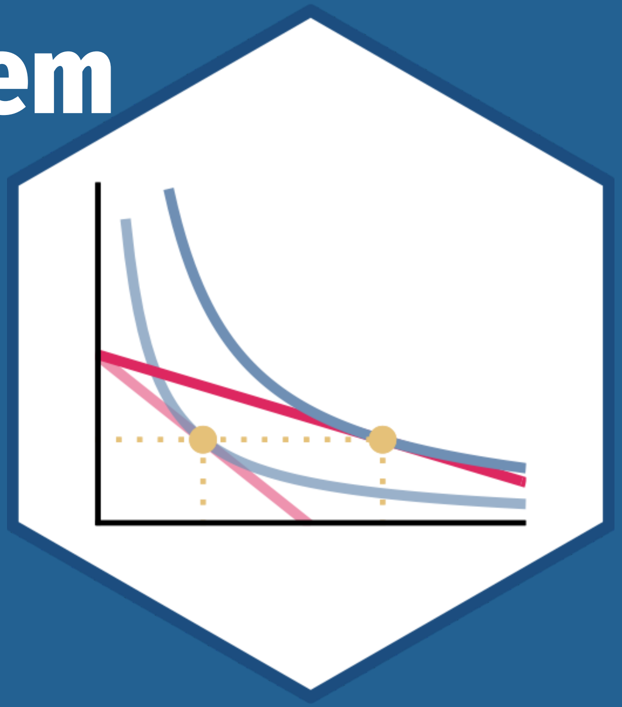


The Consumer's Problem: Review

- The **consumer's constrained optimization problem** is:

1. **Choose:** < a consumption bundle >
2. **In order to maximize:** < utility >
3. **Subject to:** < income and market prices >

The Consumer's Problem: Tools

- We now have the tools to understand consumer choices:

- Budget constraint: consumer's **constraints** of income and market prices
    - How the market trades off between two goods
- Utility function: consumer's **preferences** to maximize
    - How the consumer trades off between two goods

The Consumer's Problem: Verbally

- The **consumer's constrained optimization problem**:

> choose a bundle of goods to maximize utility, subject to income and market prices

The Consumer's Problem: Mathematically

$$\max_{x,y} u(x, y)$$

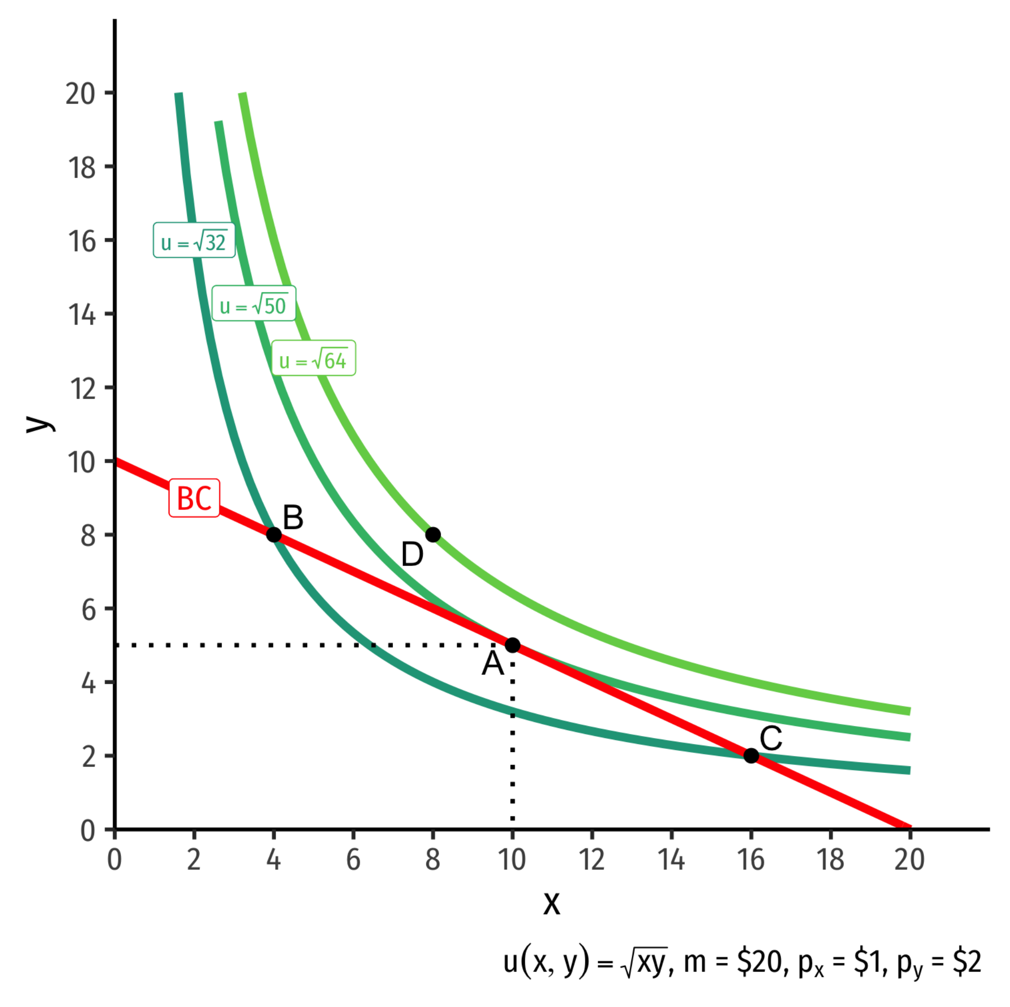
$$s.t. \; p_x x + p_y y = m$$

- This requires calculus to solve[1]. We will look at **graphs** instead!

[1] See the mathematical appendix in today's class notes on how to solve it with calculus, and an example.

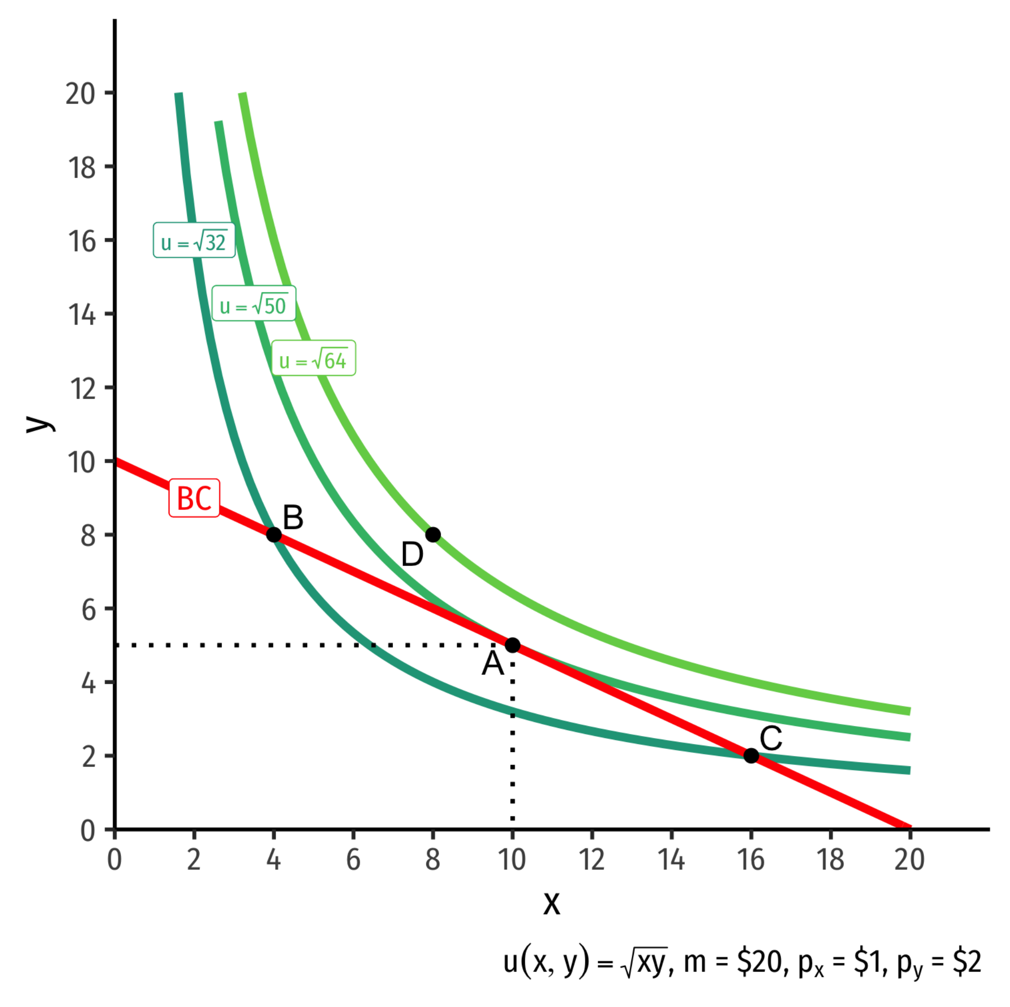
The Consumer's Optimum: Graphically

- **Graphical solution**: **Highest indifference curve *tangent* to budget constraint**
  - Bundle A!



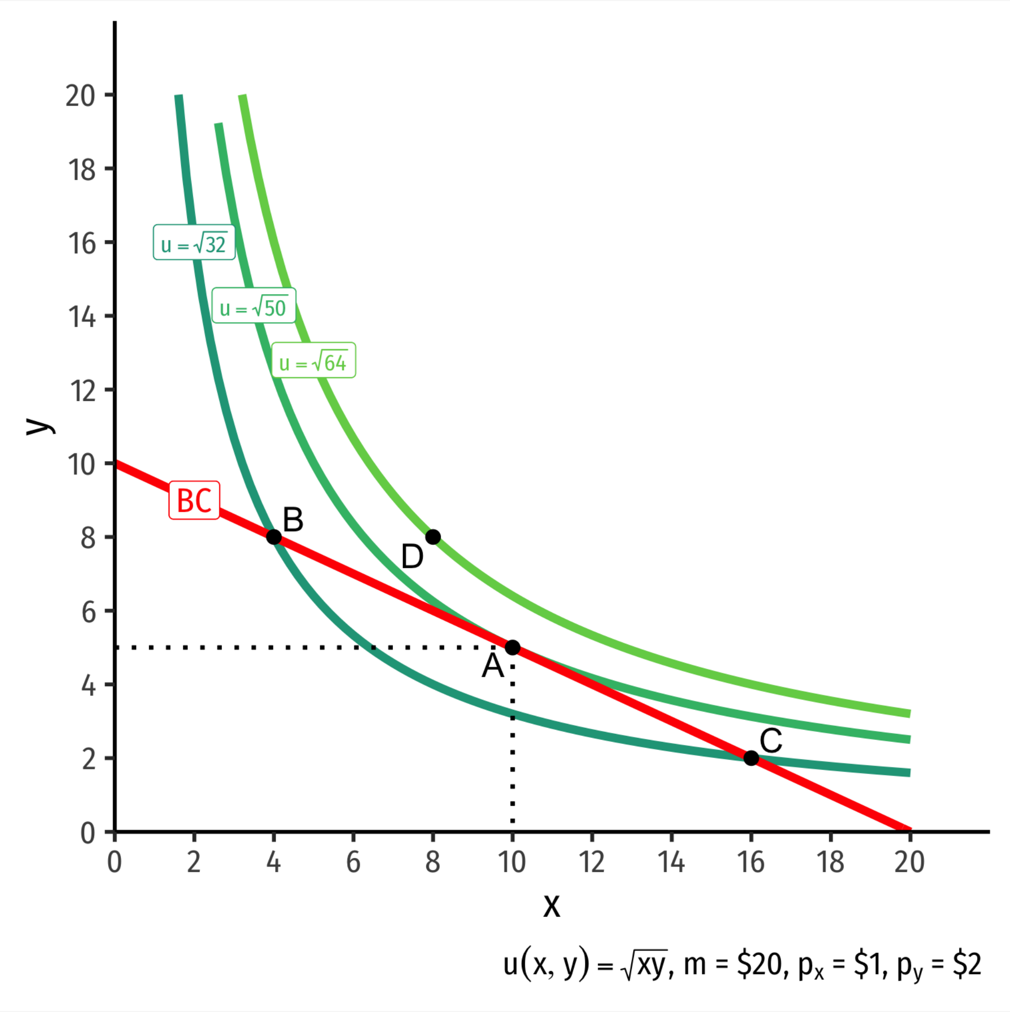
The Consumer's Optimum: Graphically

- Graphical solution: **Highest indifference curve *tangent* to budget constraint**
  - Bundle A!
- B or C spend all income, but a better combination exists
  - Averages ≻ extremes!



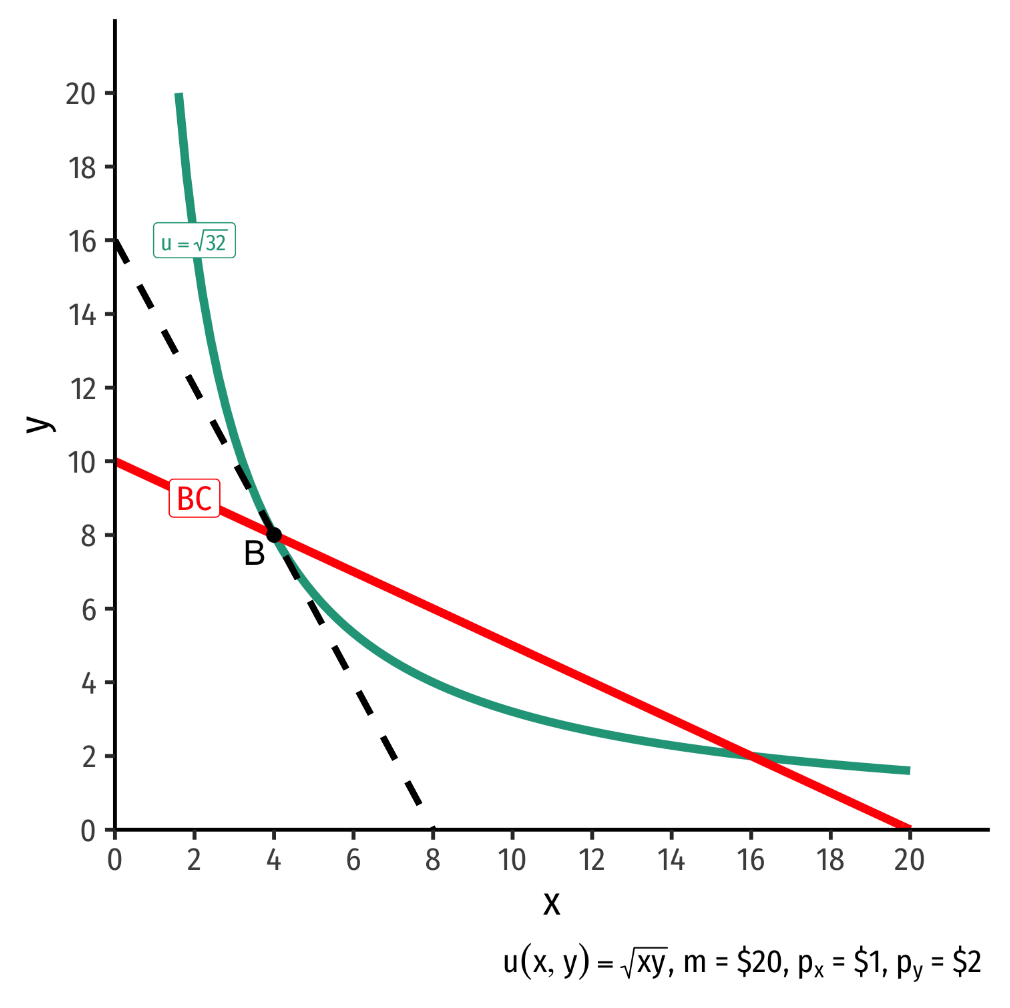
The Consumer's Optimum: Graphically

- **Graphical solution**: **Highest indifference curve *tangent* to budget constraint**
    - Bundle A!
- B or C spend all income, but a better combination exists
    - Averages $\succ$ extremes!
- D is higher utility, but *not affordable* at current income & prices



The Consumer's Optimum: Why Not B?

indiff. curve slope > budget constr. slope



The Consumer's Optimum: Why Not B?

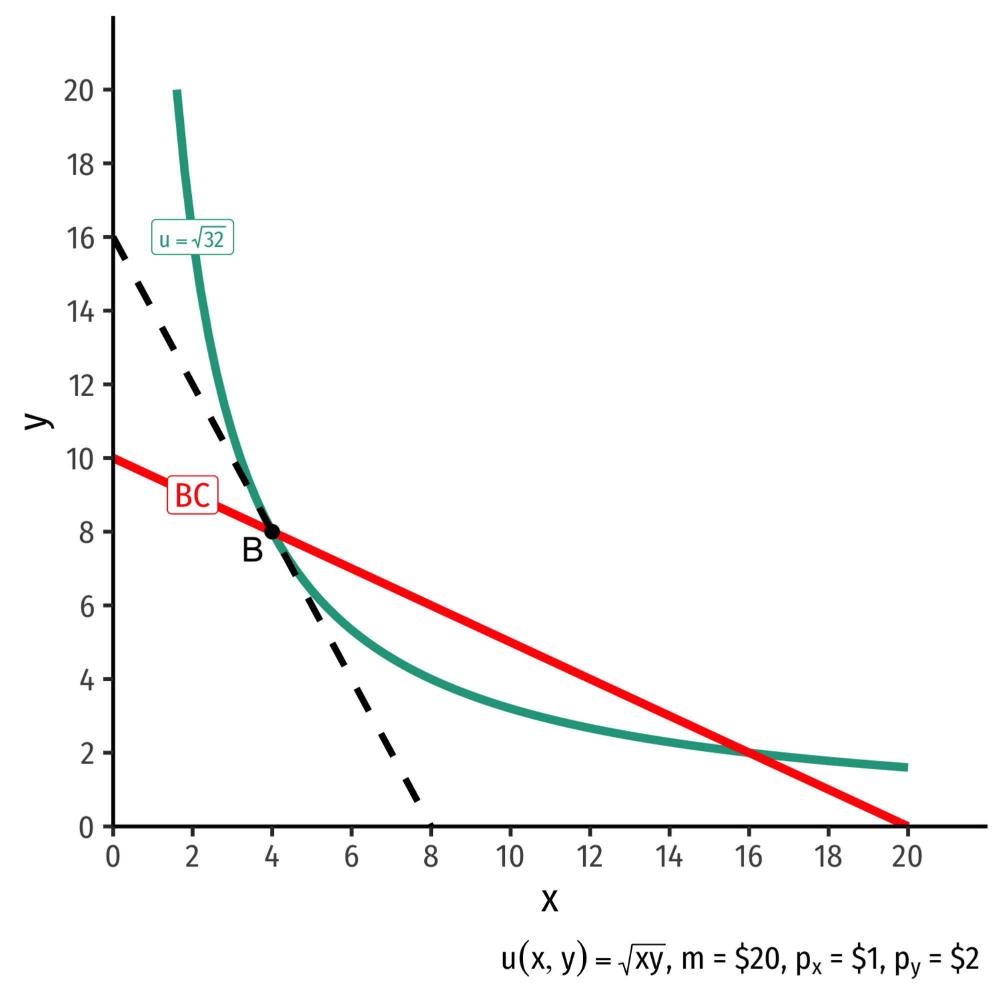
indiff. curve slope > budget constr. slope

$$|MRS_{x,y}| > |\frac{p_x}{p_y}|$$

$$|\frac{MU_x}{MU_y}| > |\frac{p_x}{p_y}|$$

$$|-2| > |-0.5|$$

- **Consumer** would exchange at **2Y:1X**
- **Market** exchange rate is **0.5Y:1X**



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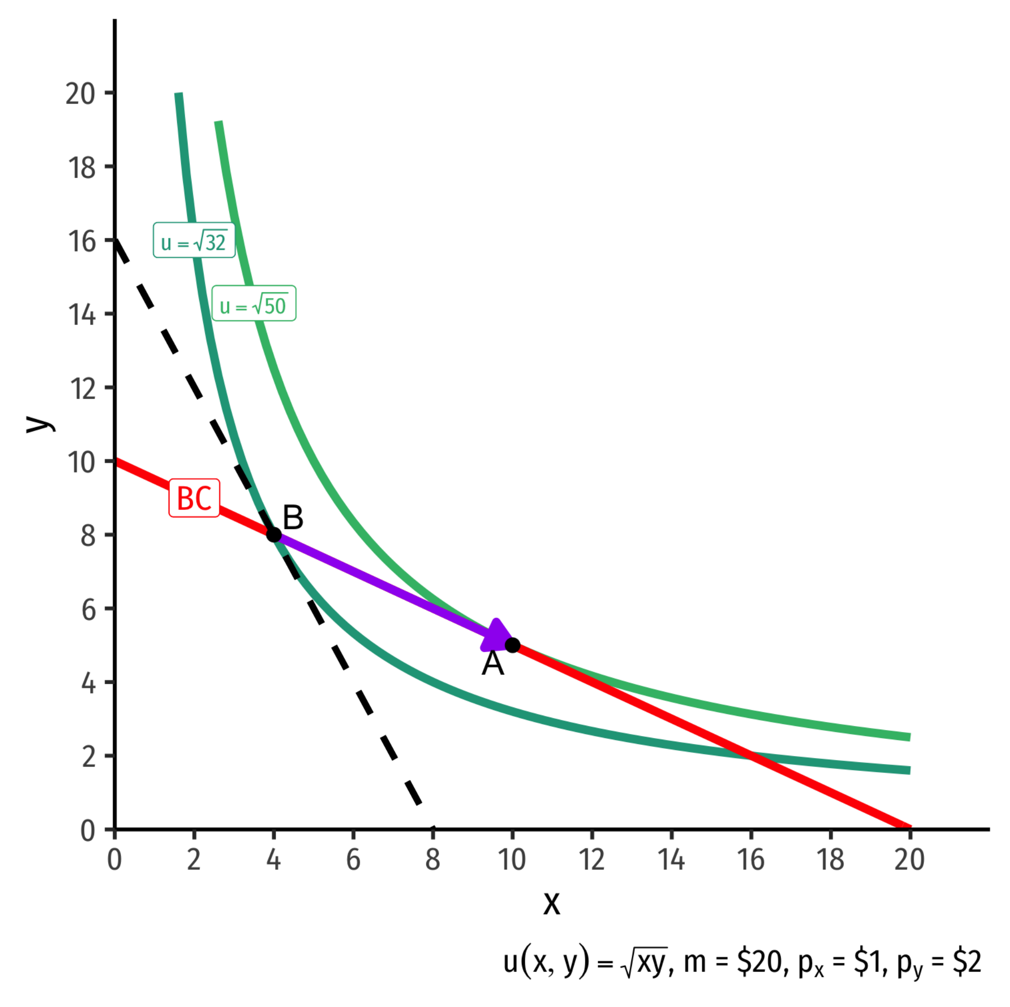
$$|MRS_{x,y}| > |\frac{p_x}{p_y}|$$

$$|\frac{MU_x}{MU_y}| > |\frac{p_x}{p_y}|$$

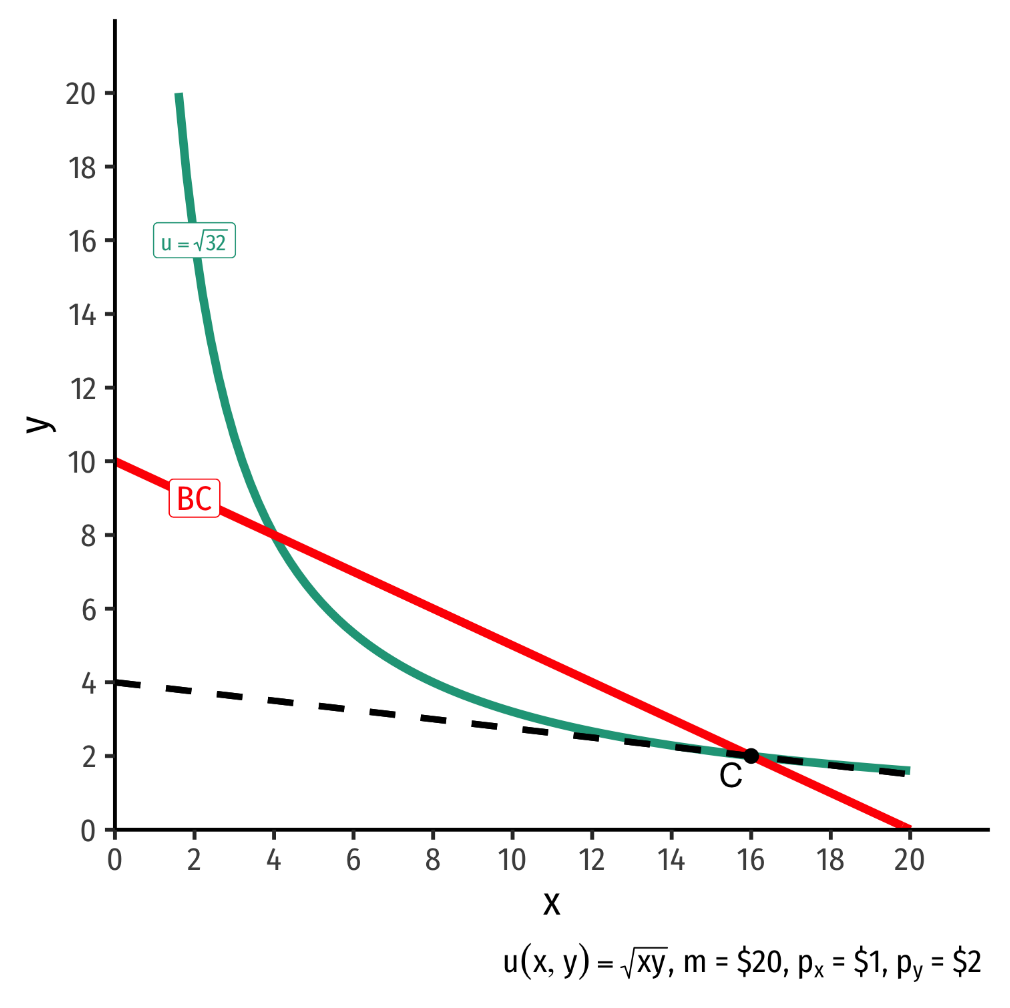
$$|-2| > |-0.5|$$

- Consumer would exchange at 2Y:1X
- Market exchange rate is 0.5Y:1X
- Can spend less on y more on x and get more utility!



The Consumer's Optimum: Why Not C?

indiff. curve slope $<$ budget constr. slope



The Consumer's Optimum: Why Not C?

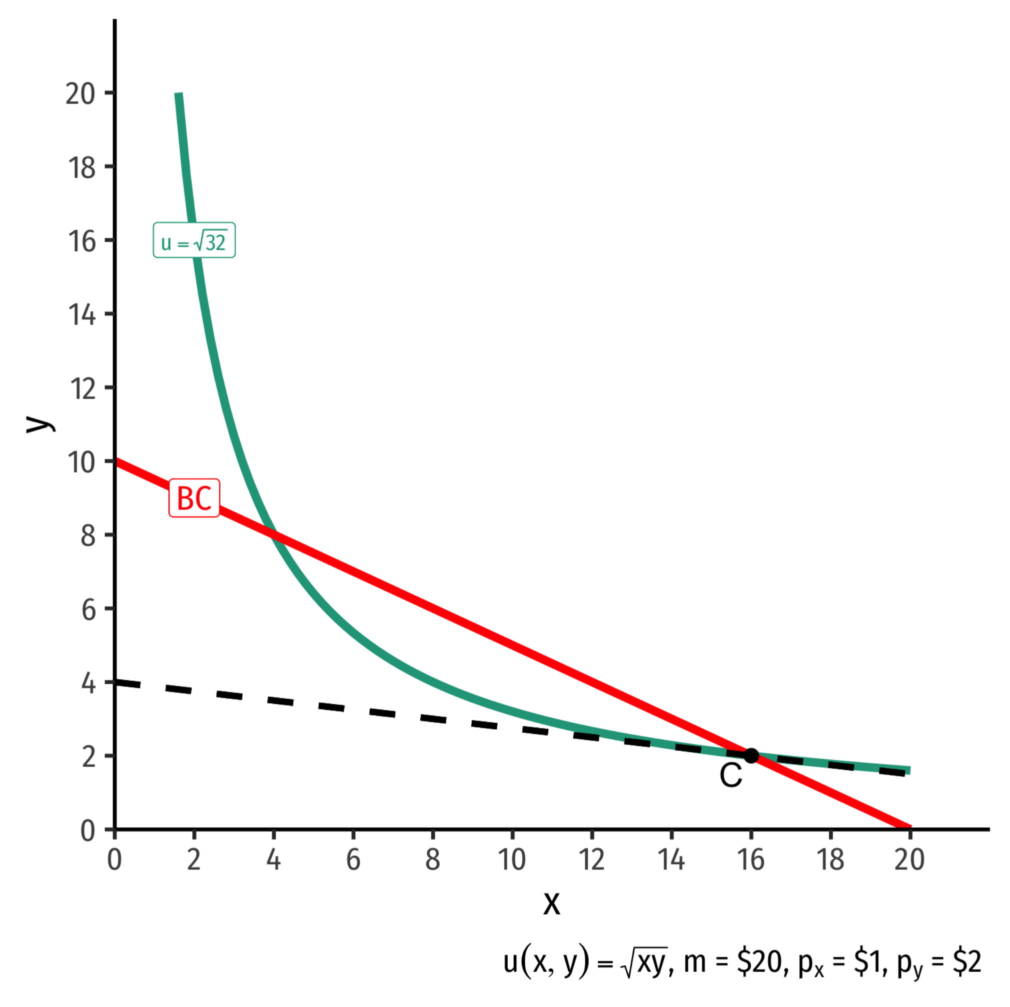
$$\text{indiff. curve slope} < \text{budget constr. slope}$$

$$|MRS_{x,y}| < |\frac{p_x}{p_y}|$$

$$|\frac{MU_x}{MU_y}| < |\frac{p_x}{p_y}|$$

$$|-0.125| < |-0.5|$$

- **Consumer** would exchange at **0.125Y:1X**
- **Market** exchange rate is **0.5Y:1X**



The Consumer's Optimum: Why Not C?

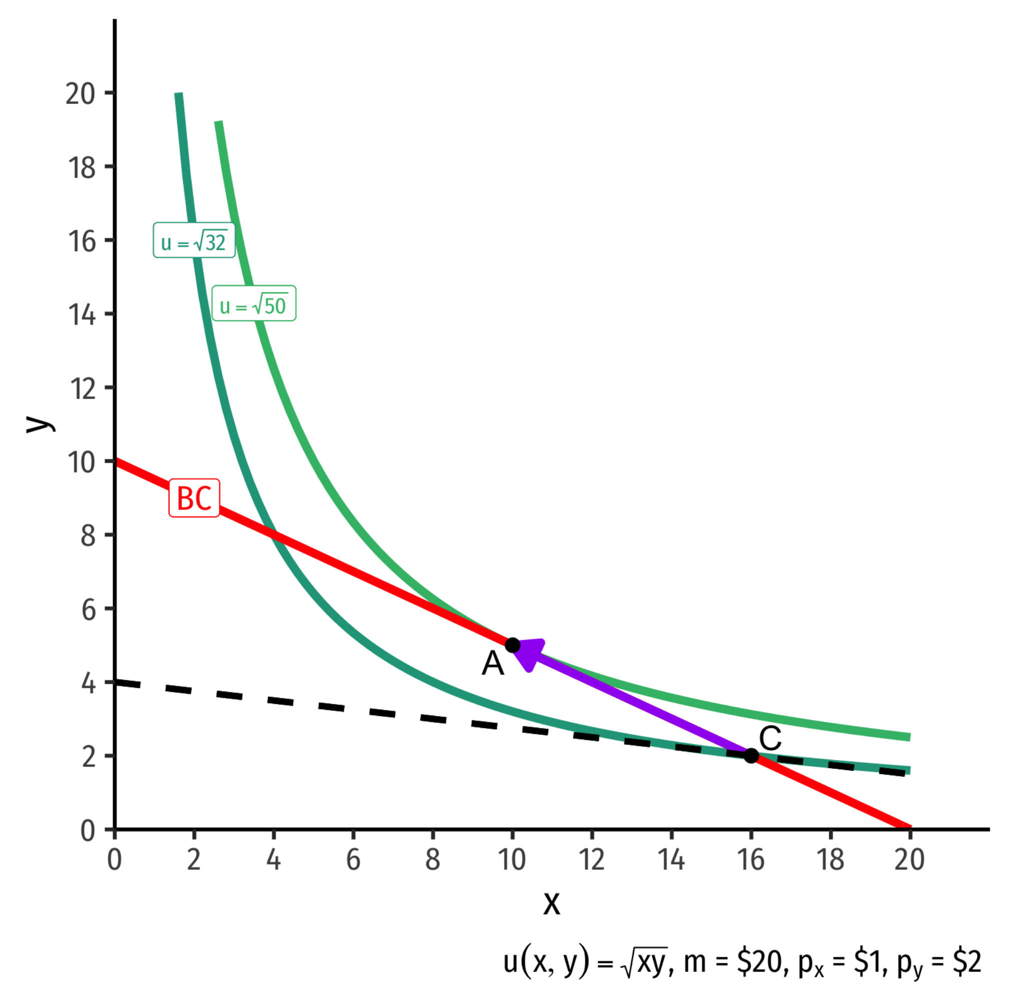
indiff. curve slope < budget constr. slope

$$|MRS_{x,y}| < |\frac{p_x}{p_y}|$$

$$|\frac{MU_x}{MU_y}| < |\frac{p_x}{p_y}|$$

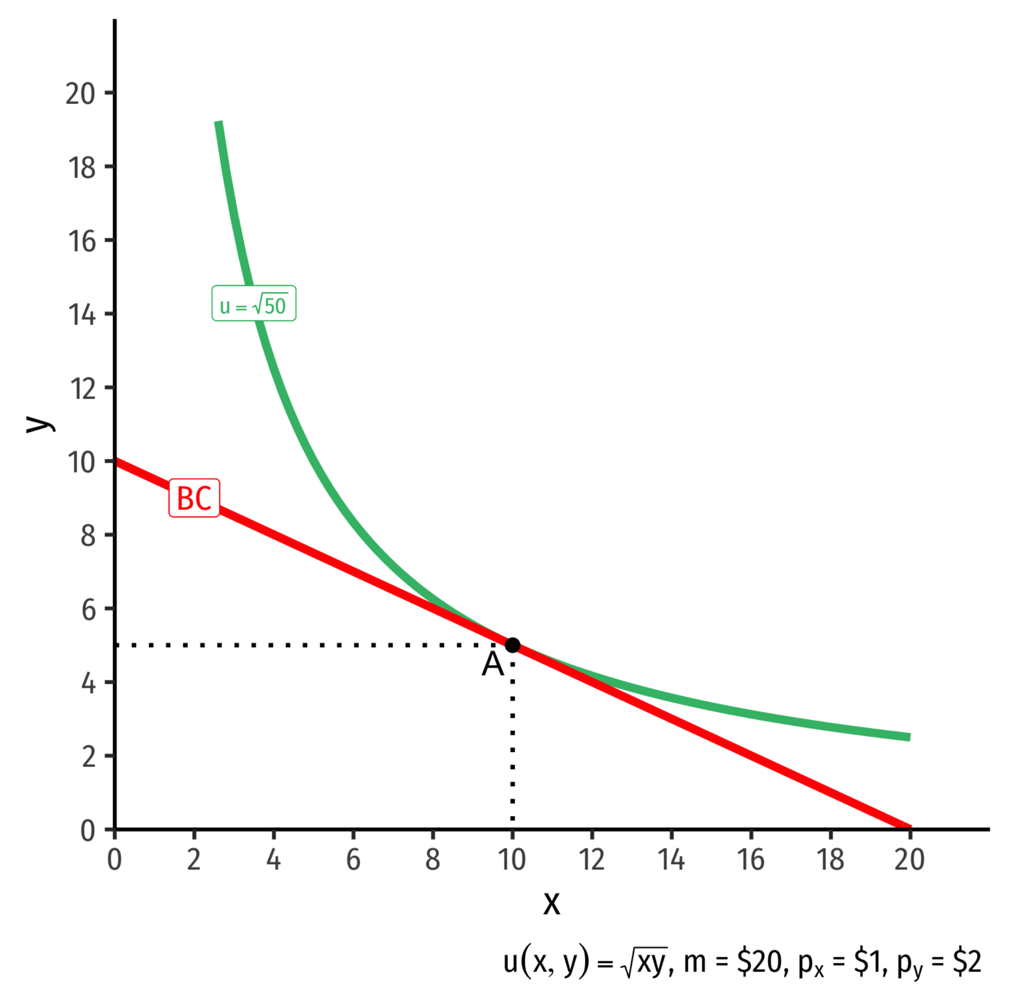
$$|-0.125| < |-0.5|$$

- **Consumer** would exchange at **0.125Y:1X**
- **Market** exchange rate is **0.5Y:1X**
- Can **spend less on x, more on y** and get **more utility!**



The Consumer's Optimum: Why A?

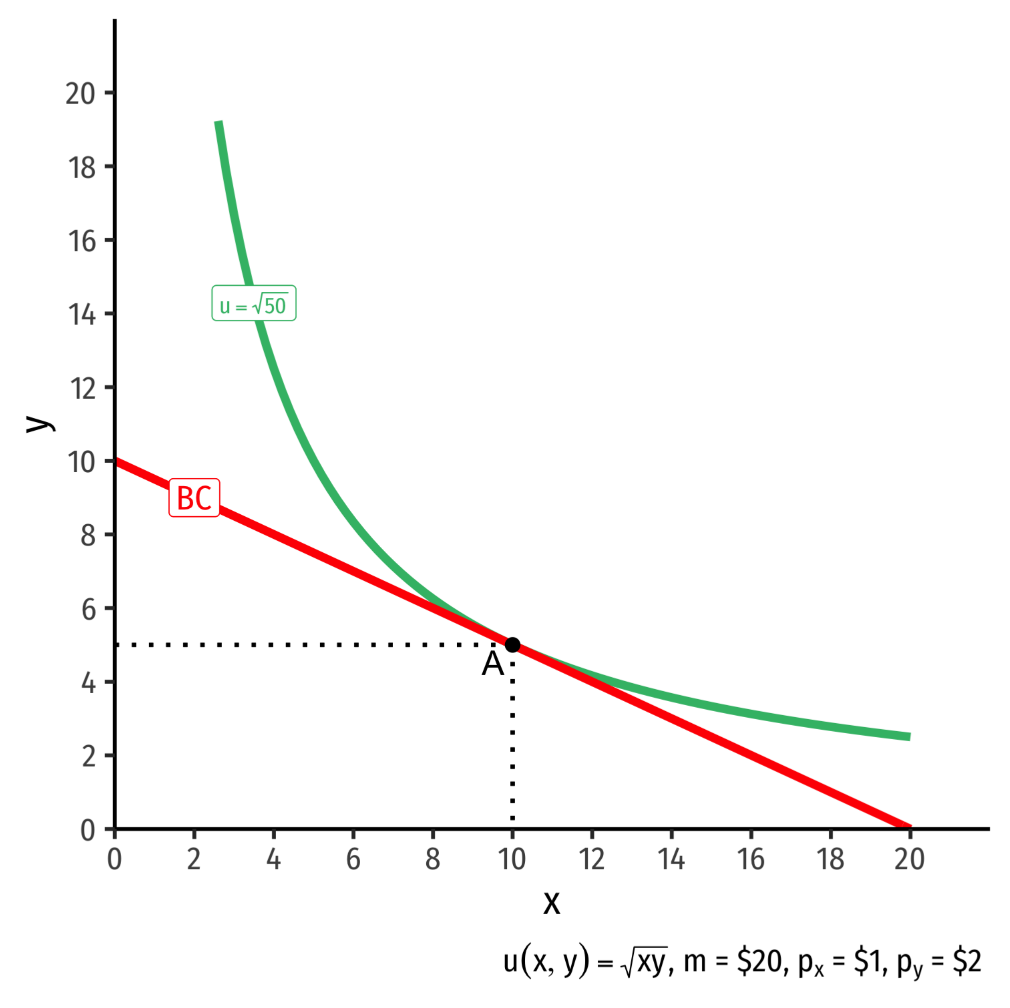
$$\text{indiff. curve slope} = \text{budget constr. slope}$$



The Consumer's Optimum: Why A?

$$\text{indiff. curve slope} = \text{budget constr. slope}$$
$$|MRS_{x,y}| = |\frac{p_x}{p_y}|$$
$$|\frac{MU_x}{MU_y}| = |\frac{p_x}{p_y}|$$
$$|-0.5| = |-0.5|$$

- **Consumer** would exchange at same rate as **market**
- ***No other combination* of (x,y) *exists* at current prices & income that could increase utility!**

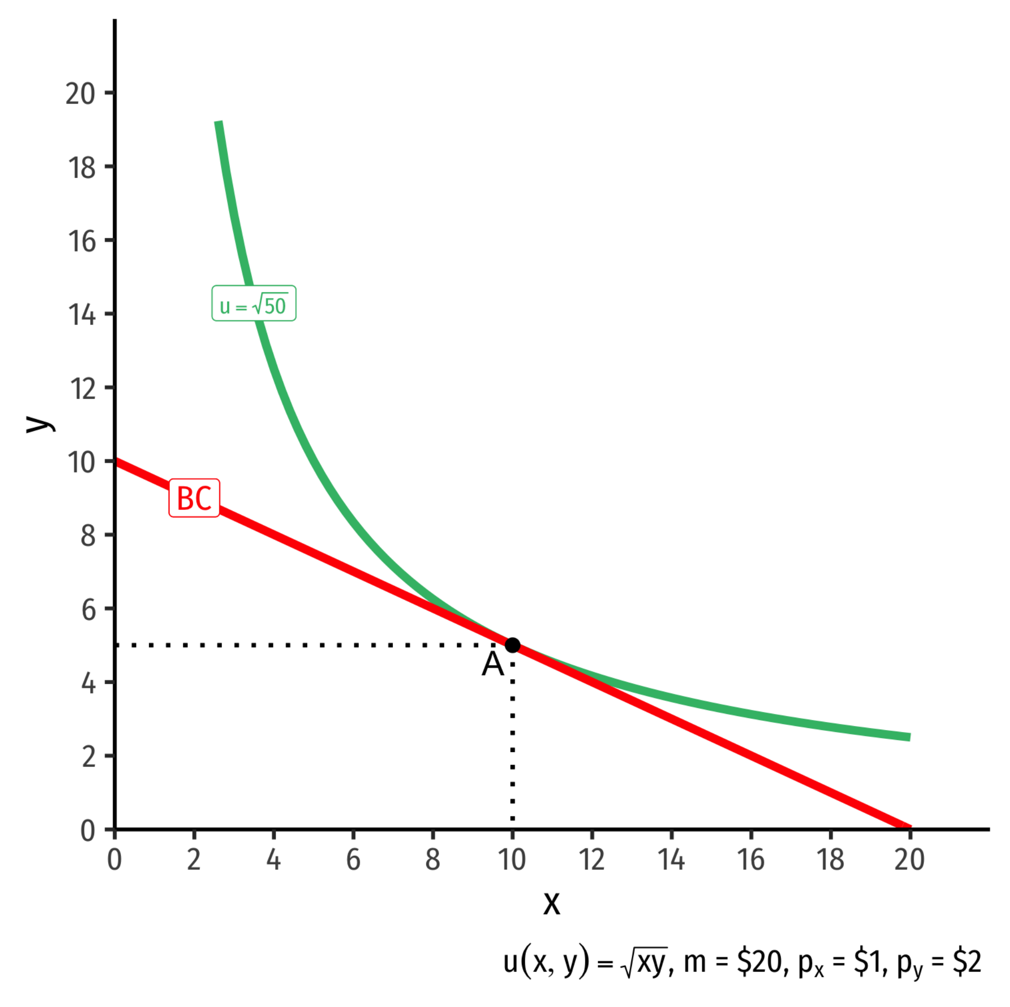


The Consumer's Optimum: Two Equivalent Rules

Rule 1

$$\frac{MU_x}{MU_y} = \frac{p_x}{p_y}$$

- Easier for calculation (slopes)



The Consumer's Optimum: Two Equivalent Rules

Rule 1

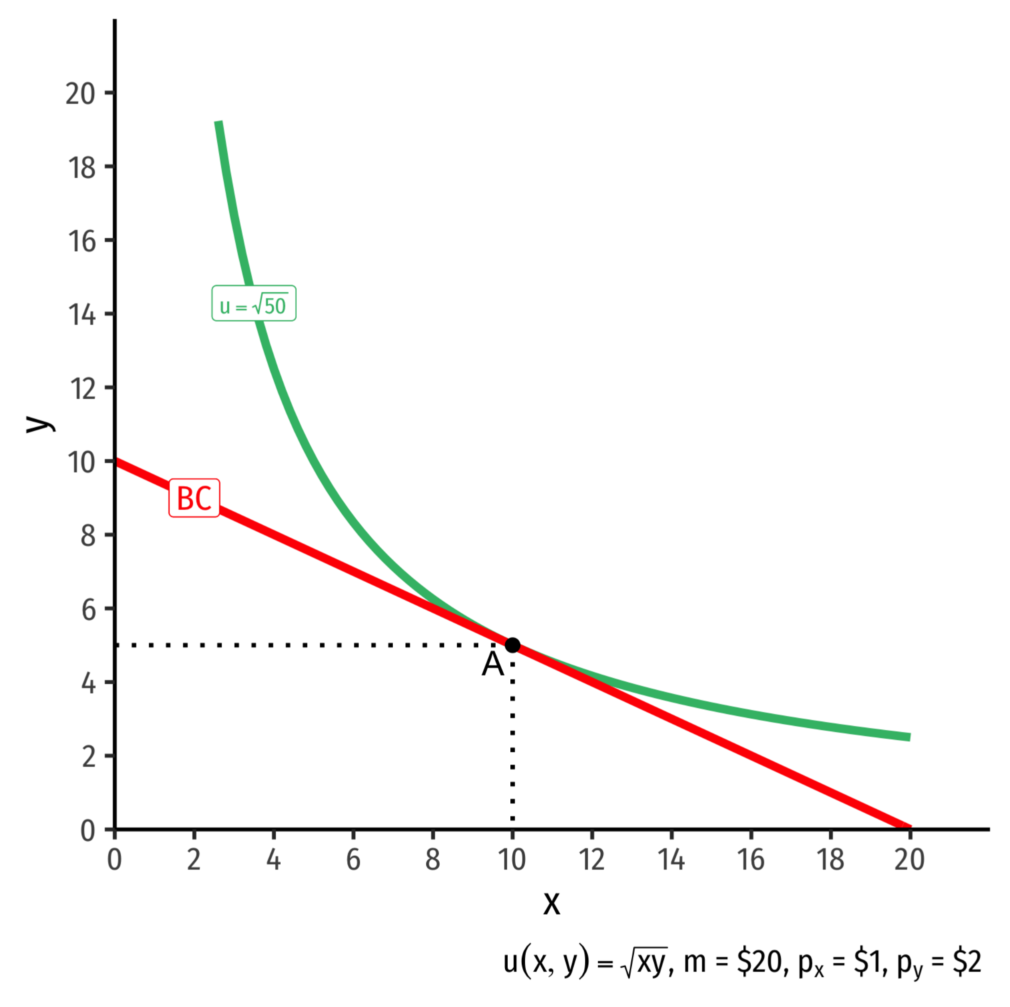
$$\frac{MU_x}{MU_y} = \frac{p_x}{p_y}$$

- Easier for calculation (slopes)

Rule 2

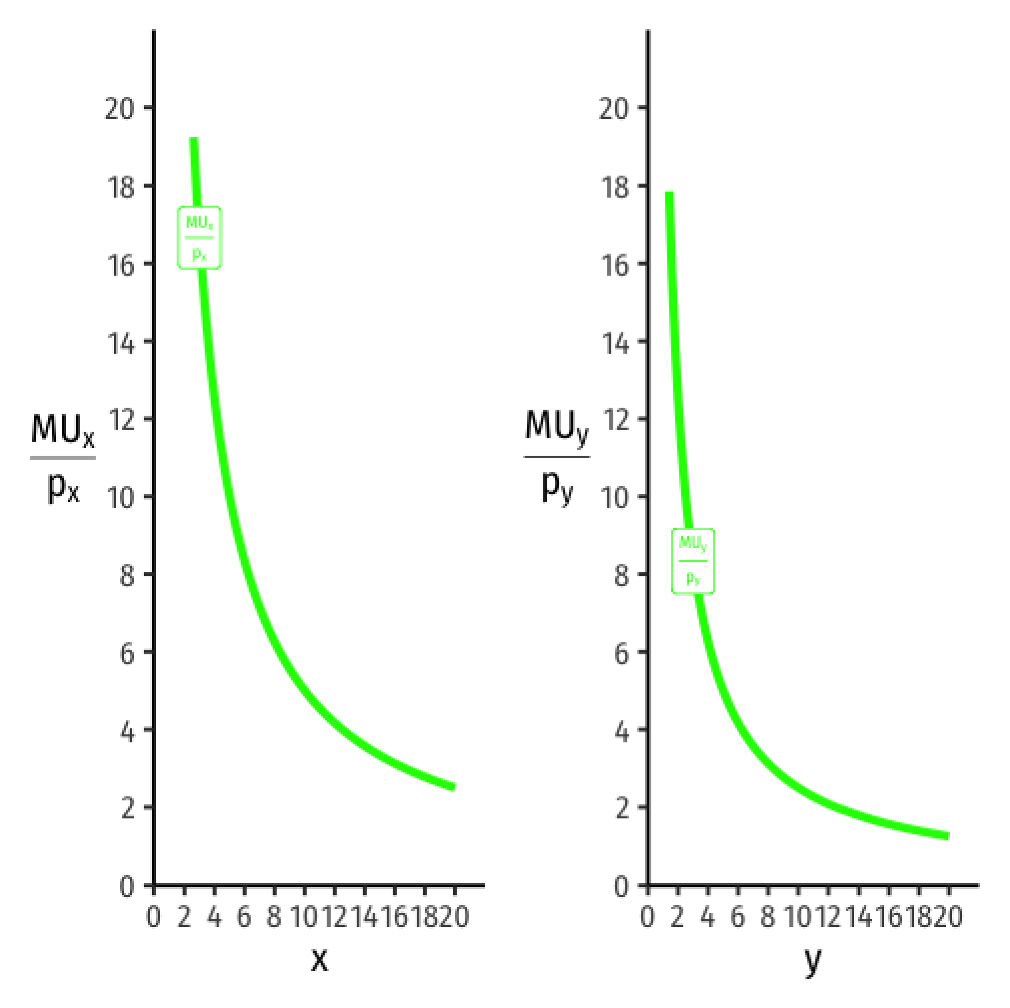
$$\frac{MU_x}{p_x} = \frac{MU_y}{p_y}$$

- Easier for intuition (next slide)



Visualizing the Equimarginal Rule

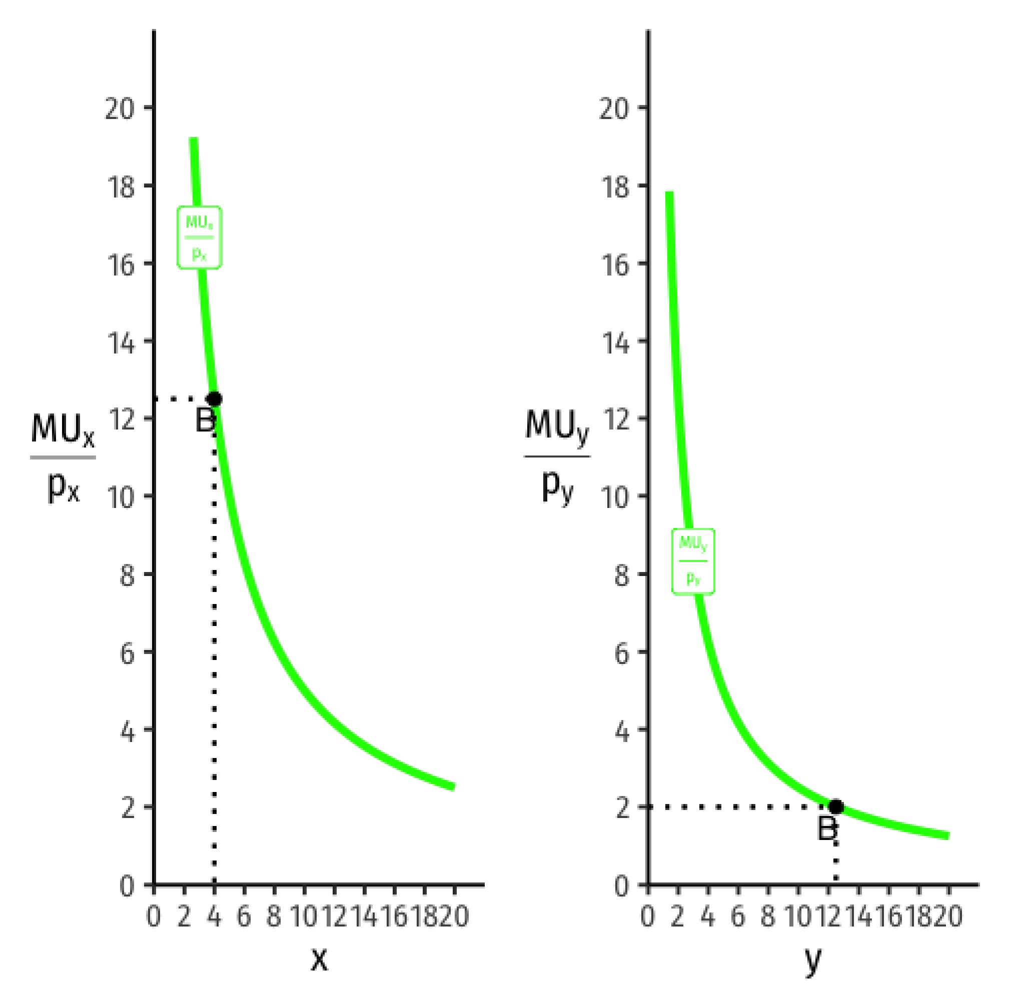
- Compare MU_x per $1 spent vs. MU_y per $1 spent
  - Graphs on right are *not* indifference curves!



Visualizing the Equimarginal Rule

- Suppose you consume 4 of x and 12.5 of y (points B)

$$\frac{MU_x}{p_x} > \frac{MU_y}{p_y}$$



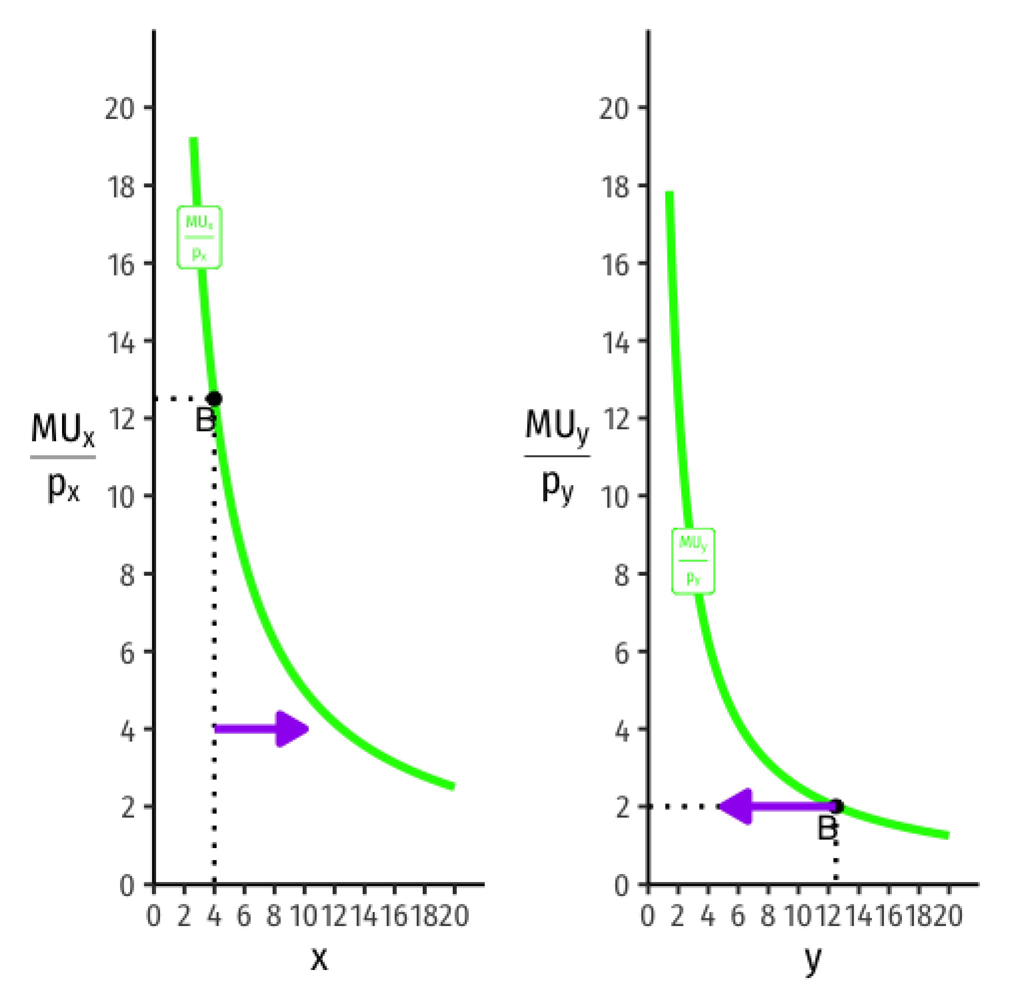
Visualizing the Equimarginal Rule

- Suppose you consume 4 of x and 12.5 of y (points B)

$$\frac{MU_x}{p_x} > \frac{MU_y}{p_y}$$

- More "bang for your buck" with x than y

- Consume more x, less y!



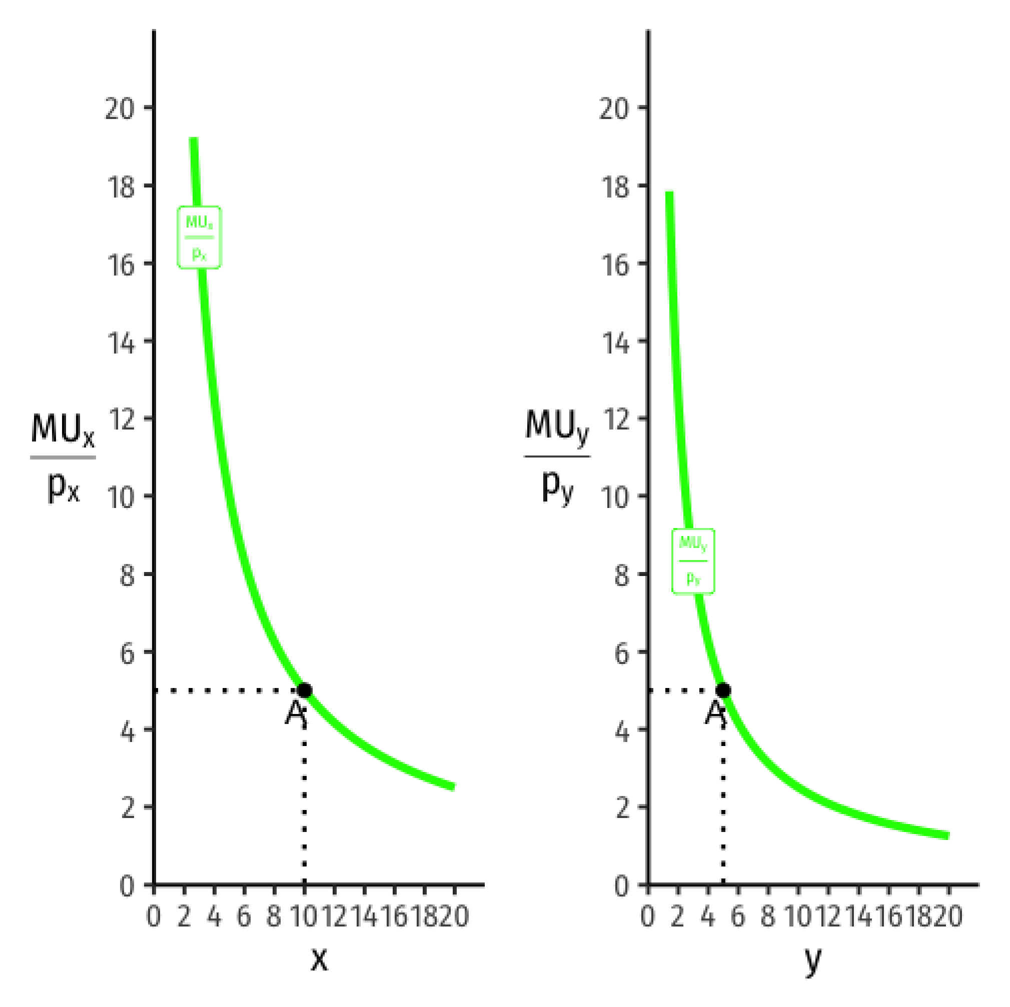
Visualizing the Equimarginal Rule

- At points A, consuming 10 of x and 5 of y

$$\frac{MU_x}{p_x} = \frac{MU_y}{p_y}$$

- No change (more x, less x, more y, less y) that could increase your utility!

- The optimum! Cost-adjusted marginal utilities are equalized



The Consumer's Optimum: The Equimarginal Rule I

$$\frac{MU_x}{p_x} = \frac{MU_y}{p_y} = \cdots = \frac{MU_n}{p_n}$$

- **Equimarginal Rule**: consumption is optimized where the **marginal utility per dollar spent** is **equalized** across all n possible goods/decisions

- You will always choose an option that gives higher marginal utility (e.g. if $MU_x > MU_y$)
  - But each option has a different cost, so we weight each option by its cost, hence $\frac{MU_x}{p_x}$

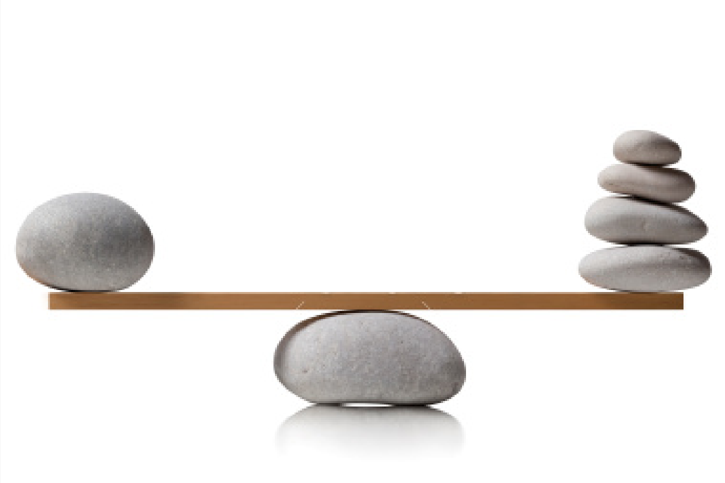
The Consumer's Optimum: The Equimarginal Rule II

- Any **optimum** in economics: no better alternatives exist under current constraints

- No possible change in your consumption that would increase your utility

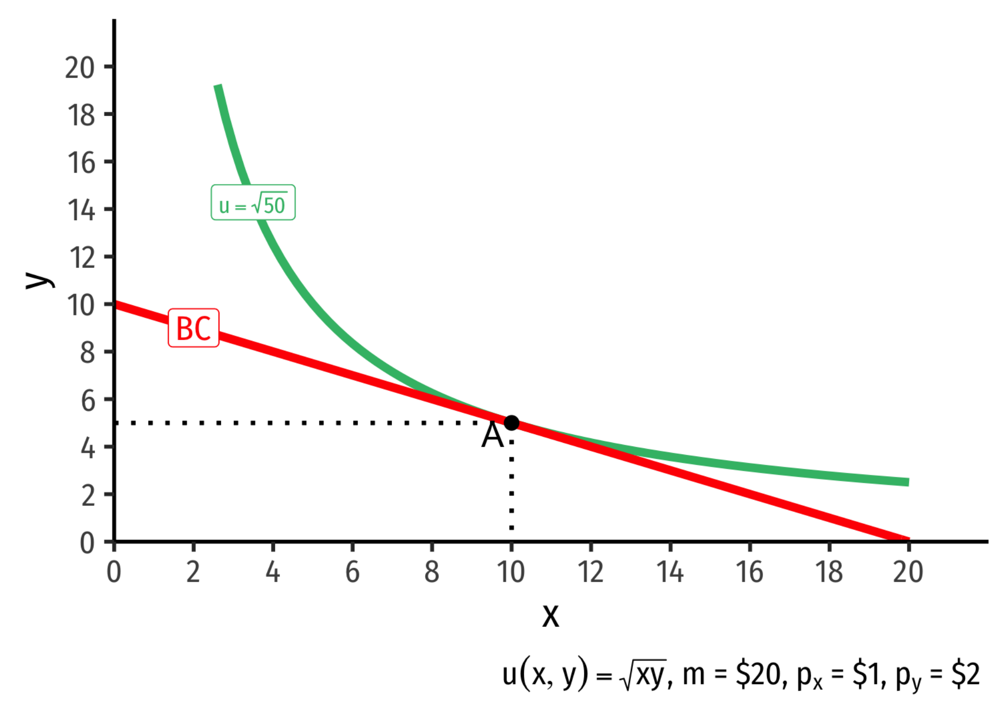
Markets Equalize Everyone's MRS I

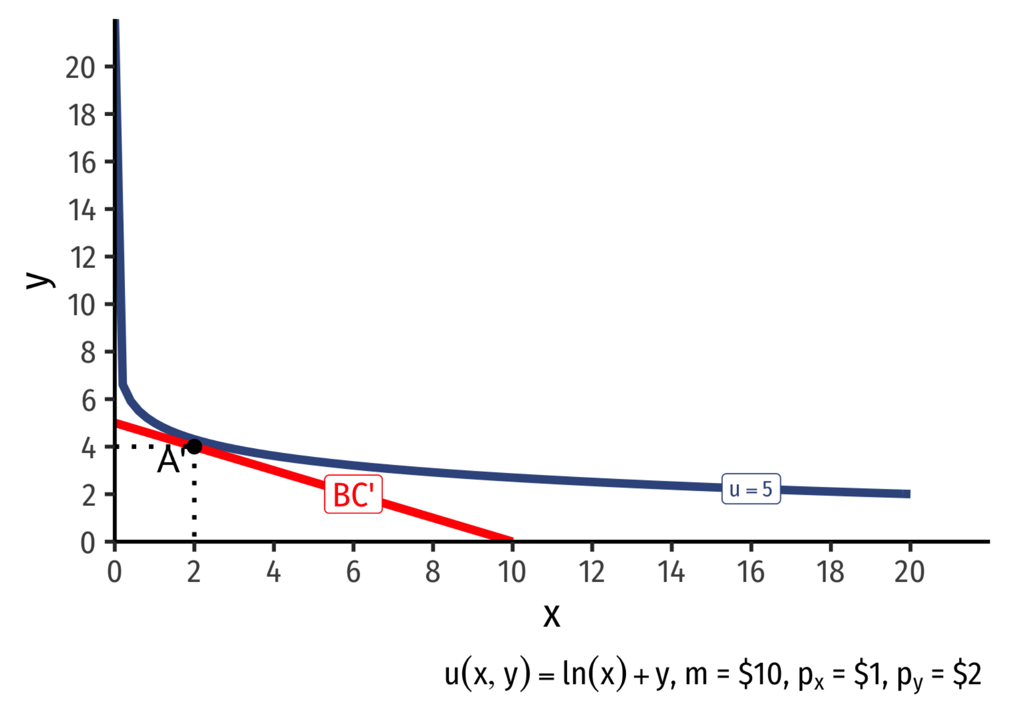
- **Markets make it so everyone faces the *same* relative prices**
  - Budget constraint. slope, $-\frac{p_x}{p_y}$
  - Note individuals' incomes, m, are certainly different!
- A person's optimal choice $\implies$ they make same tradeoff as the market
  - Their MRS $=$ relative price ratio
- **markets equalize everyone's MRS**



Markets Equalize Everyone's MRS II

Two people will very different income and preferences face the same market prices, and choose optimal consumption (points A and A') at an exchange rate of $0.5Y : 1X$





Optimization and Equilibrium

- If people can *learn* and *change* their behavior, they will always switch to a higher-valued option

- If a person has no *better* choices (under current constraints), they are at an **optimum**

- **If everyone is at an optimum**, the **system** is in **equilibrium**

Practice I

Example: You can get utility from consuming bags of Almonds (a) and bunches of Bananas (b), according to the utility function:

$$u(a, b) = ab$$
$$MU_a = b$$
$$MU_b = a$$

You have an income of $50, the price of Almonds is $10, and the price of Bananas is $2. Put Almonds on the horizontal axis and Bananas on the vertical axis.

1. What is your utility-maximizing bundle of Almonds and Bananas?
2. How much utility does this provide? [Does the answer to this matter?]

Practice II, Cobb-Douglas!

Example: You can get utility from consuming Burgers (b) and Fries (f), according to the utility function:

$$
\begin{aligned}
u(b,f) &= \sqrt{bf} \\
MU_b &= 0.5b^{-0.5}f^{0.5} \\
MU_f &= 0.5b^{0.5}f^{-0.5}
\end{aligned}
$$

You have an income of $20, the price of Burgers is $5, and the price of Fries is $2. Put Burgers on the horizontal axis and Fries on the vertical axis.

1. What is your utility-maximizing bundle of Burgers and Fries?
2. How much utility does this provide?