

1.7 — Income & Substitution Effects

ECON 306 • Microeconomic Analysis • Fall 2020

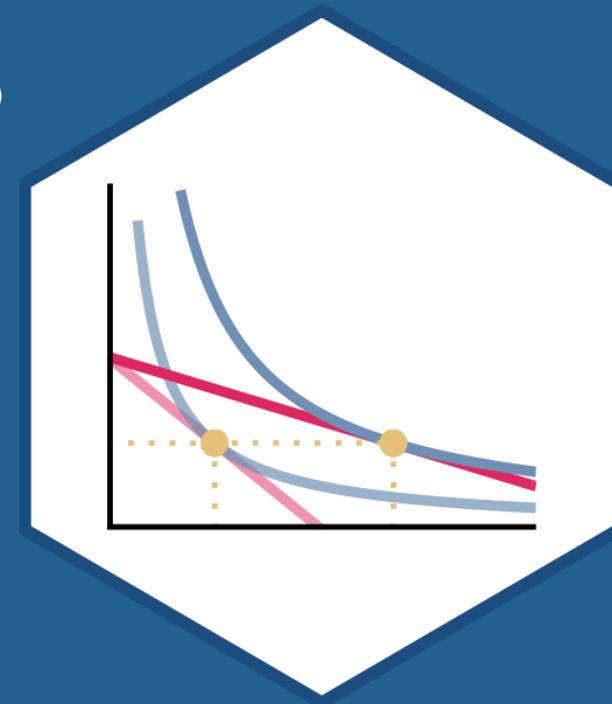
Ryan Safner

Assistant Professor of Economics

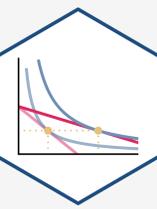
[✉ safner@hood.edu](mailto:safner@hood.edu)

[🔗 ryansafner/microF20](https://github.com/ryansafner/microF20)

[🌐 microF20.classes.ryansafner.com](https://microF20.classes.ryansafner.com)



A Demand Function (Again)

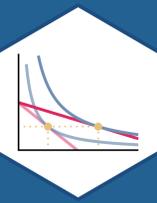


- A consumer's **demand** (for good x) depends on current prices & income:

$$q_x^D = q_x^D(m, p_x, p_y)$$

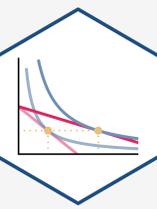
- How does **demand for x** change?
 1. **Income effects** $\left(\frac{\Delta q_x^D}{\Delta m}\right)$: how (q_x^D) changes with changes in income
 2. **Cross-price effects** $\left(\frac{\Delta q_x^D}{\Delta p_y}\right)$: how (q_x^D) changes with changes in prices of *other* goods (e.g. (y))
 3. **(Own) Price effects** $\left(\frac{\Delta q_x^D}{\Delta p_x}\right)$: how (q_x^D) changes with changes in price (of (x))





The (Own) Price Effect

The (Own) Price Effect



- **Price effect:** change in optimal consumption of a good associated with a change in its price, holding income and other prices constant

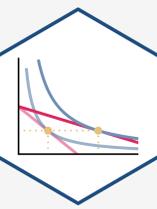
$$\frac{\Delta q_x^D}{\Delta p_x} < 0$$

The law of demand: as the price of a good rises, people will tend to buy less of that good (and vice versa)

- i.e. **the price effect is negative!**



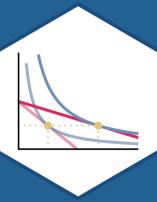
Decomposing the Price Effect



The **price effect** (law of demand) is actually the **net result of two effects**

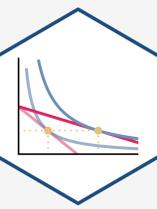
1. **(Real) income effect**: change in consumption due to change in real purchasing power
2. **Substitution effect**: change in consumption due to change in relative prices

Price Effect \ (= \) **Real income effect** \ (+ \) **Substitution Effect**

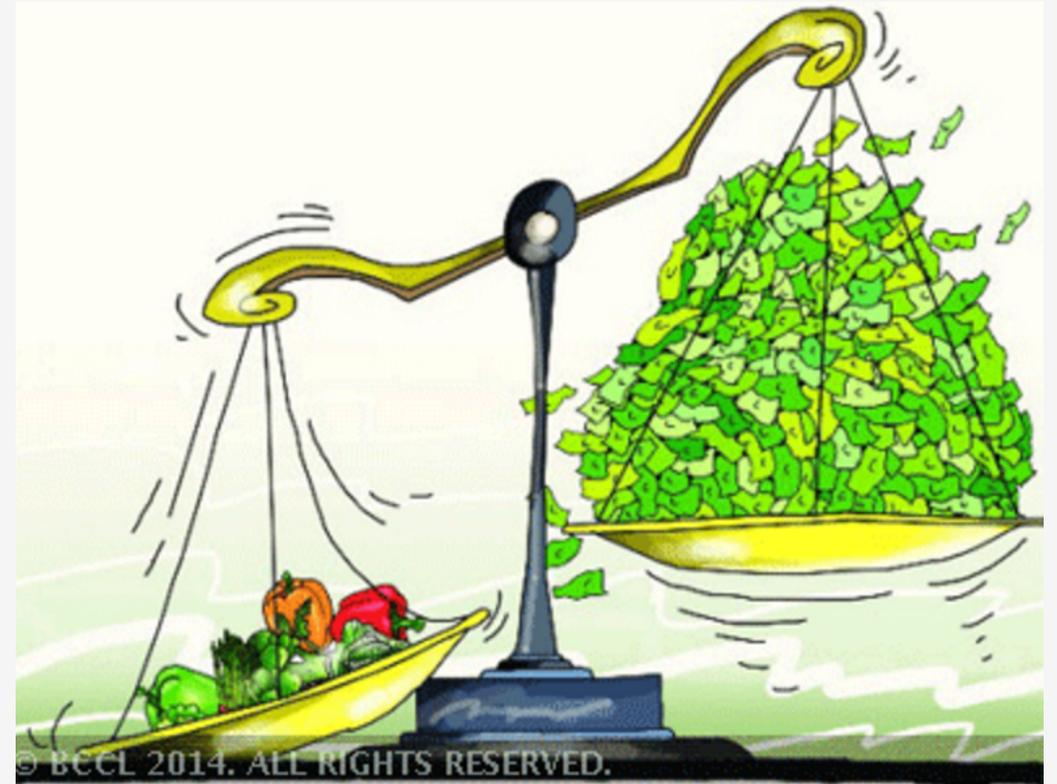


(Real) Income Effect

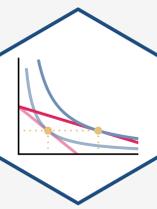
(Real) Income Effect: Demonstration



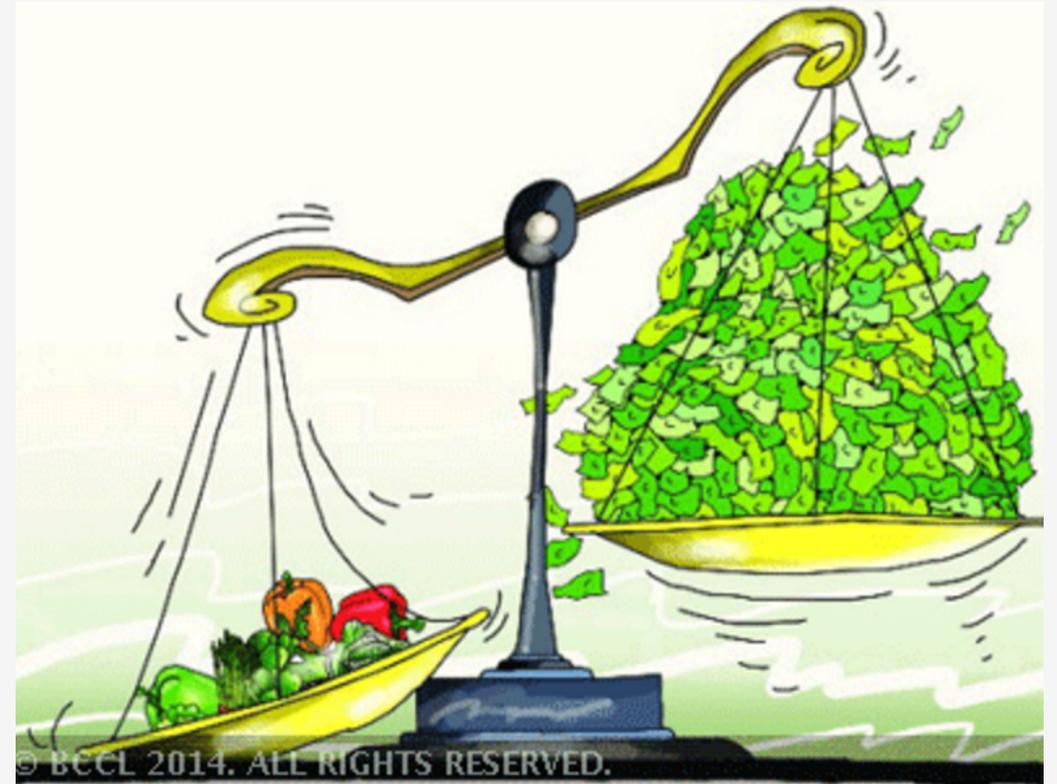
- Suppose there is only 1 good to consume, x . You have a \$100 income, and the price of x is \$10. You consume 10 units of x
- Suppose the price of x falls to \$5. You now consume 20 units of x .
- This is the **real income effect**



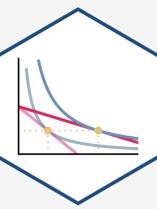
(Real) Income Effect: Demonstration



- **Real income effect:** your consumption mix changes because of the change in the price of x changes your **real income** or **purchasing power** (the amount of goods you can buy)
- Note your **actual (nominal) income** (\$100) **never changed!**



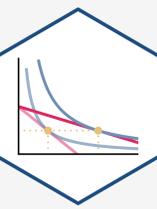
(Real) Income Effect: Size



- The *size* of the income effect depends on how large a *portion of your budget* you spend on the good
- **Large-budget items:**
 - e.g. Housing/apartment rent, car prices
 - Price increase makes you much poorer
 - Price decrease makes you much wealthier

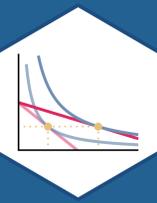


(Real) Income Effect: Size



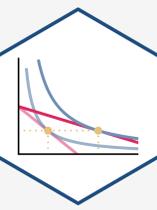
- The *size* of the income effect depends on how large a *portion of your budget* you spend on the good
- **Small-budget items:**
 - e.g. pencils, toothpicks, candy
 - Price changes don't have much of an effect on your wealth or change your behavior much





Substitution Effect

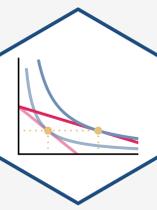
Substitution Effect: Demonstration



- Suppose there are 1000's of goods, none of them a major part of your budget
 - So real income effect is insignificant
- Suppose the price of one good, x , increases
- You would consume *less* of x relative to other goods because x is now *relatively* more expensive
- That's the **substitution effect**

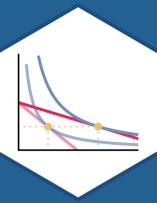


Substitution Effect: Demonstration



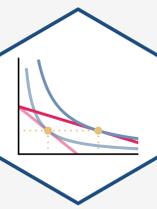
- **Substitution effect**: consumption mix changes because of a change in **relative prices**
- Buy more of the (now) relatively cheaper items
- Buy less of the (now) relatively more expensive item $((x))$





Putting the Effects Together

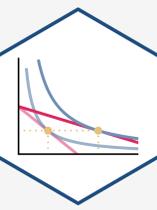
Putting the Effects Together



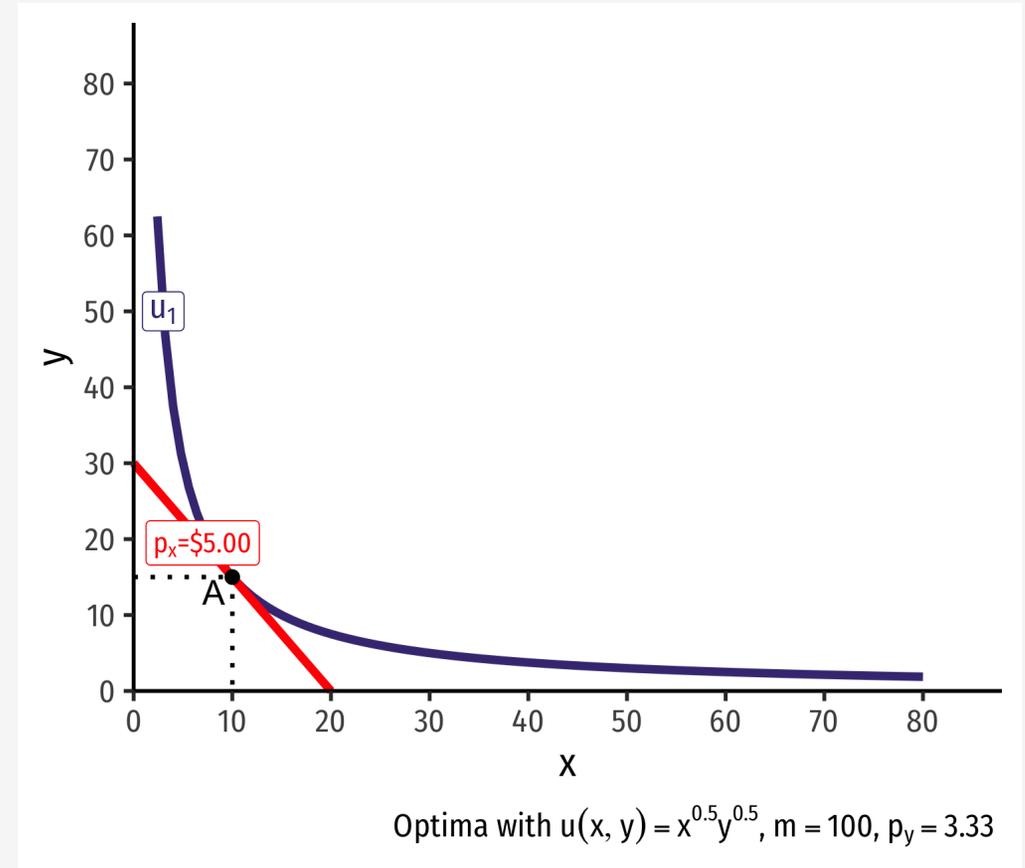
- **Real income effect:** change in consumption due to change in real purchasing power
 - Can be positive (**normal goods**) or negative (**inferior goods**)
 - Lower price of x means you can buy more x , y , or *both* (depending on your preferences between x and y)
- **Substitution effect:** change in consumption due to change in relative prices
 - If x gets cheaper relative to y , consume $\downarrow y$ (and $\uparrow x$)
 - This is always the same direction! \downarrow relatively expensive goods, \uparrow relatively cheaper goods
 - This is why demand curves slope downwards!

Price Effect \downarrow **Real income effect** \downarrow **Substitution Effect**

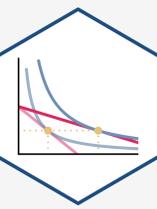
Real Income and Substitution Effects, Graphically I



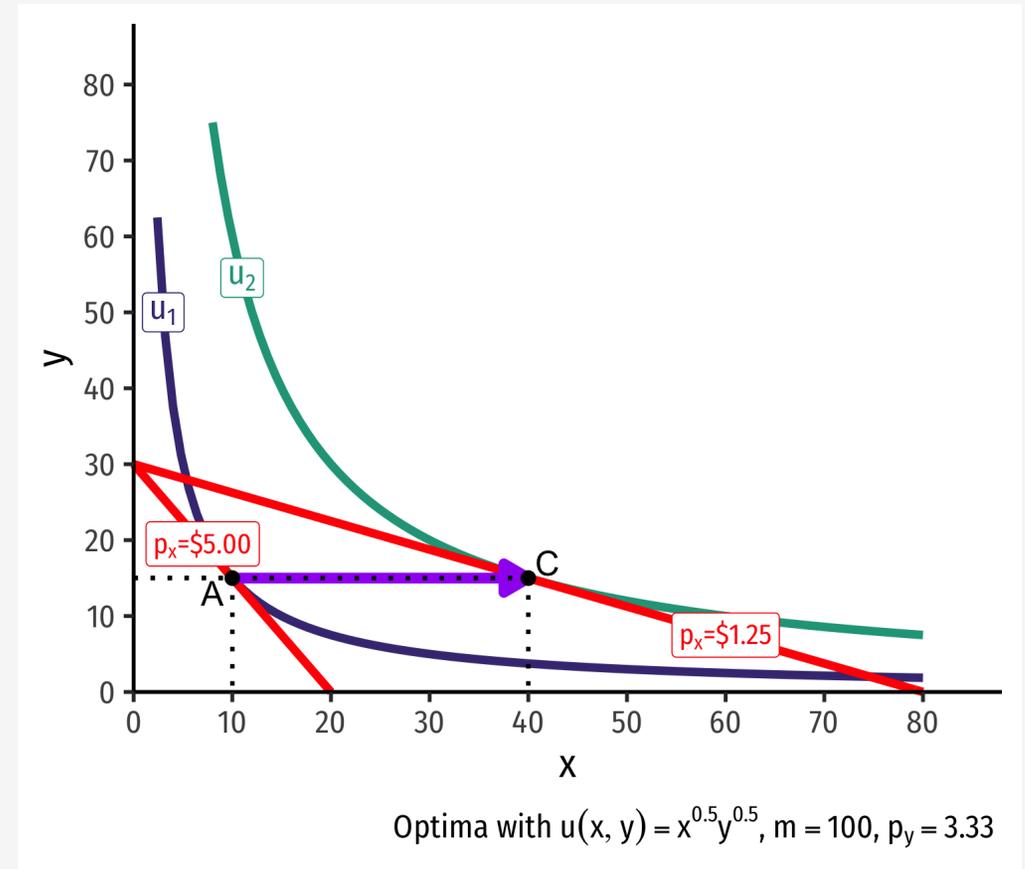
- Original optimal consumption $((A))$



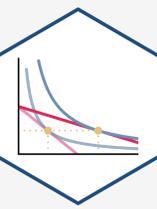
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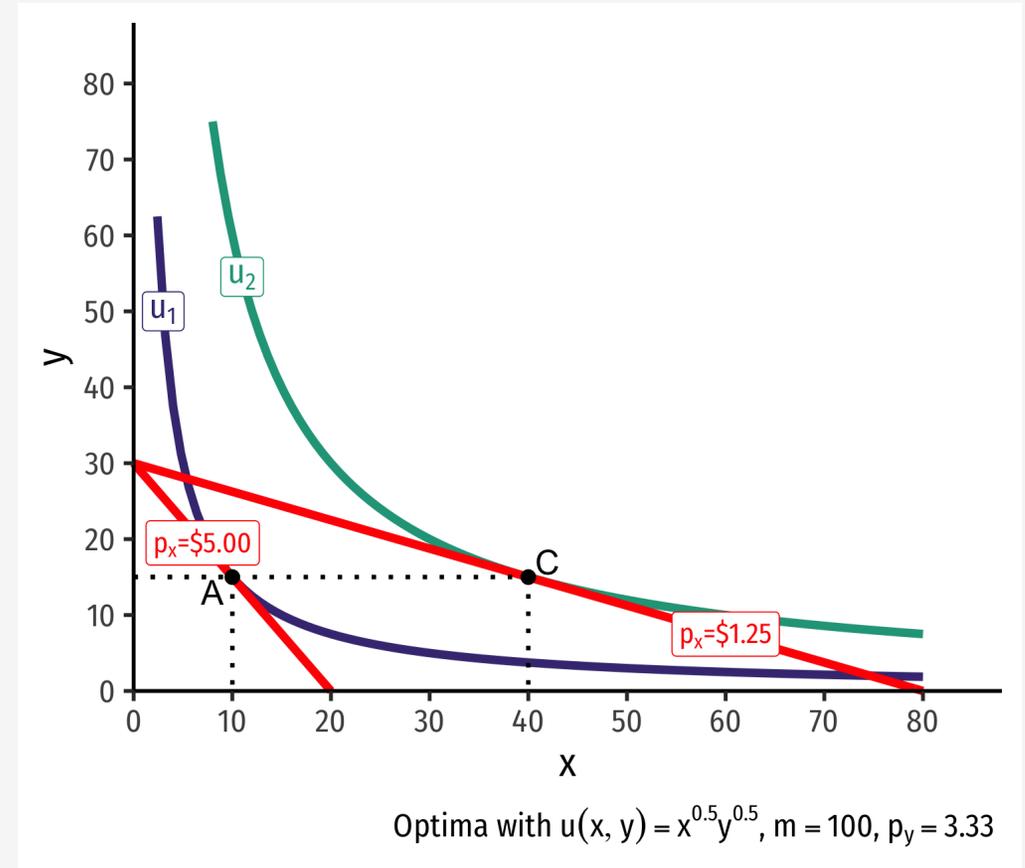
- Original optimal consumption (A)
- **(Total) price effect:** $(A \rightarrow C)$
- Let's decompose this into the two effects



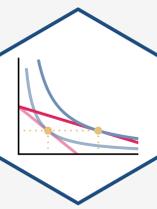
Real Income and Substitution Effects, Graphically II



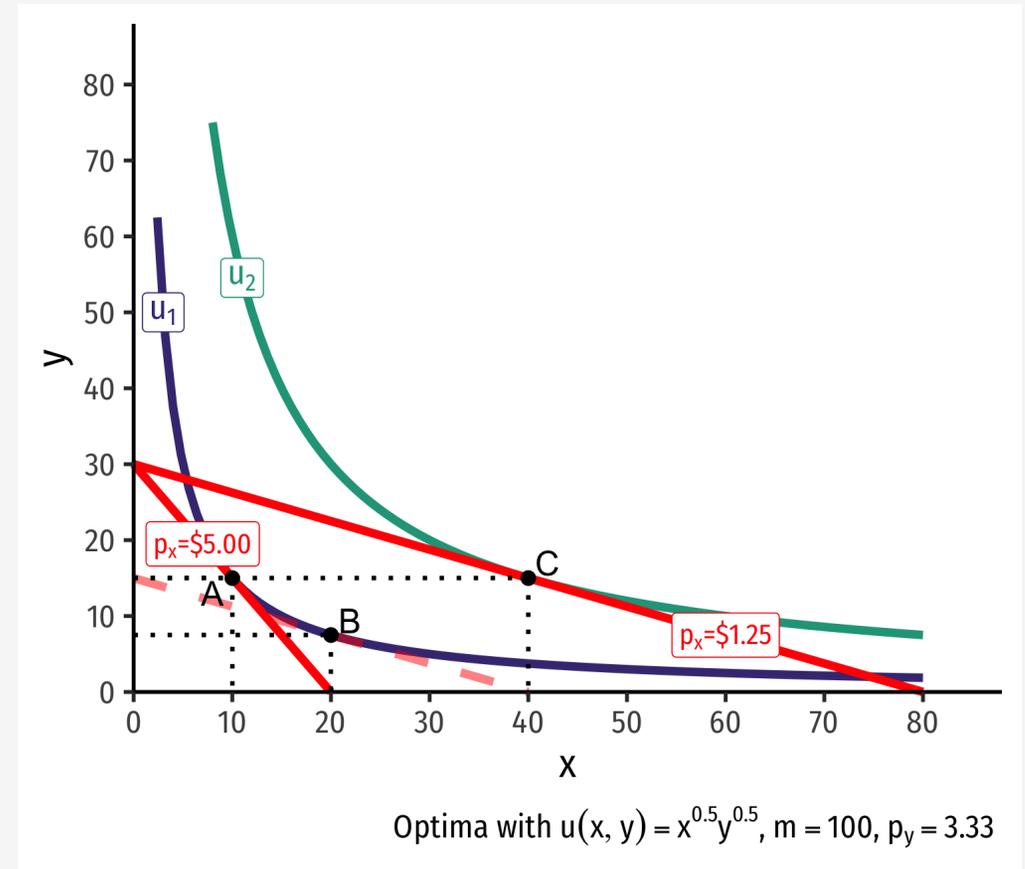
- **Substitution effect:** what you would choose under the **new exchange rate** to **remain indifferent** as before the change



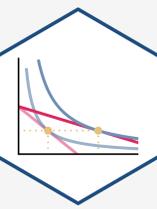
Real Income and Substitution Effects, Graphically II



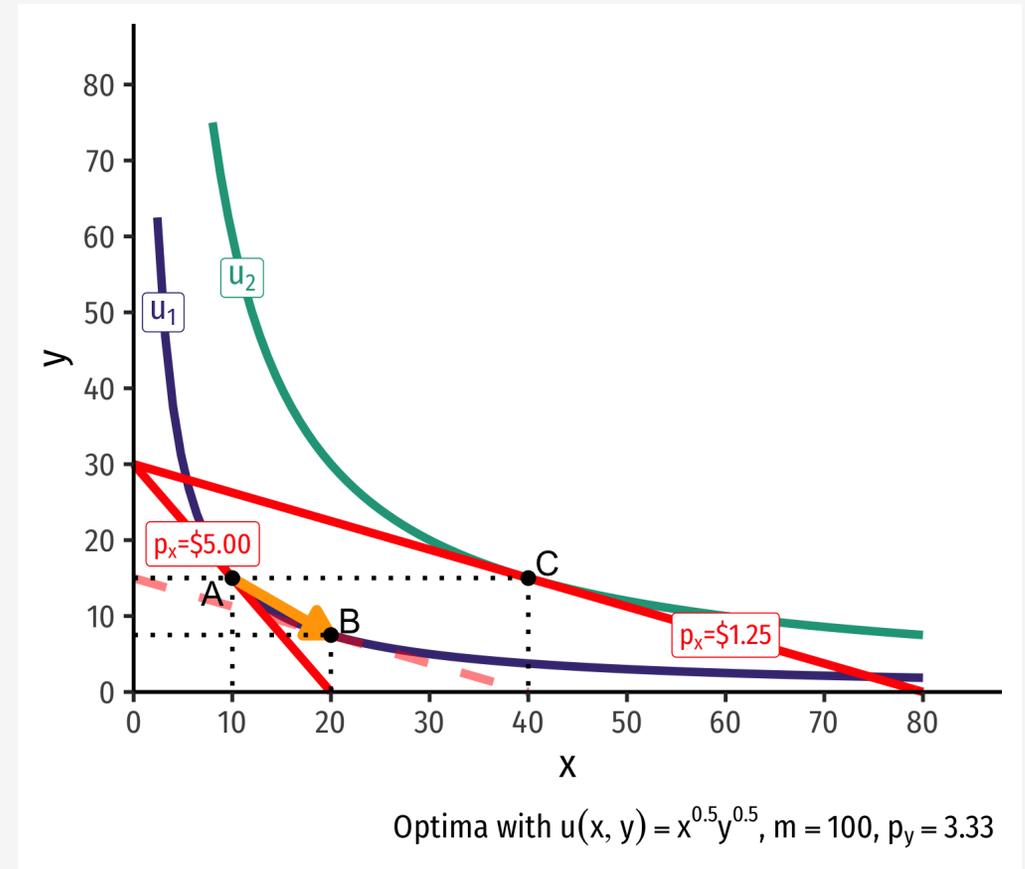
- **Substitution effect:** what you would choose under the **new exchange rate** to **remain indifferent** as before the change
- Graphically: shift *new* budget constraint inwards until tangent with *old* indifference curve
- $(A \rightarrow B)$ on same I.C. $(\uparrow x, \downarrow y)$
 - Point B *must* be a *different* point on the original curve!



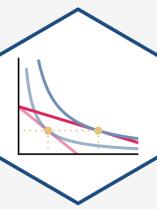
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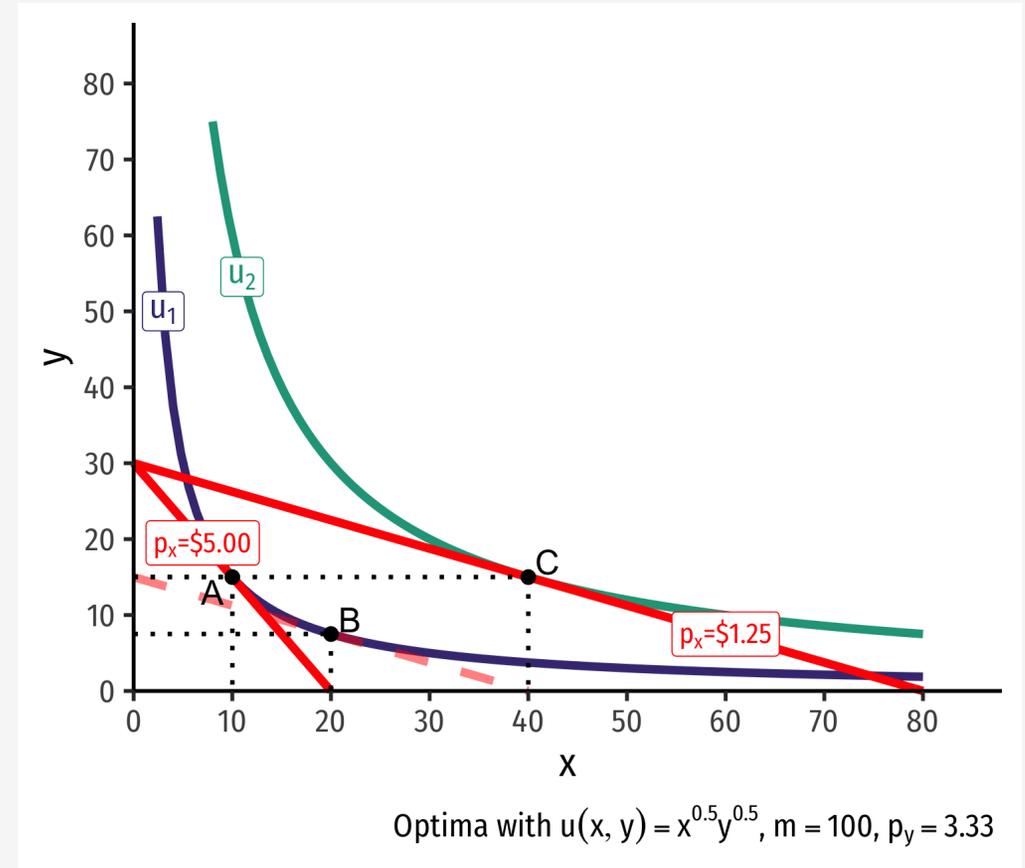
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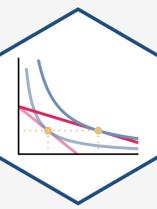
Real Income and Substitution Effects, Graphically III



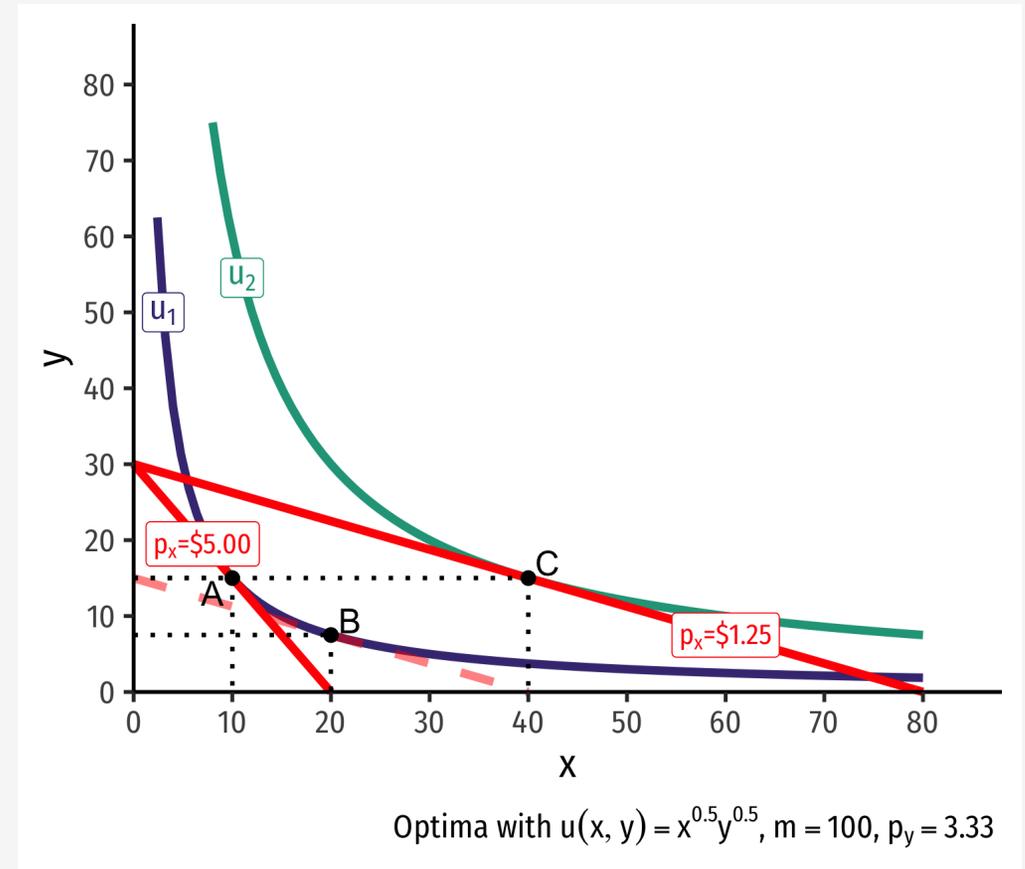
- **(Real) income effect:** change in consumption due to the **change in purchasing power** from the change in price



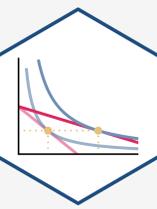
Real Income and Substitution Effects, Graphically III



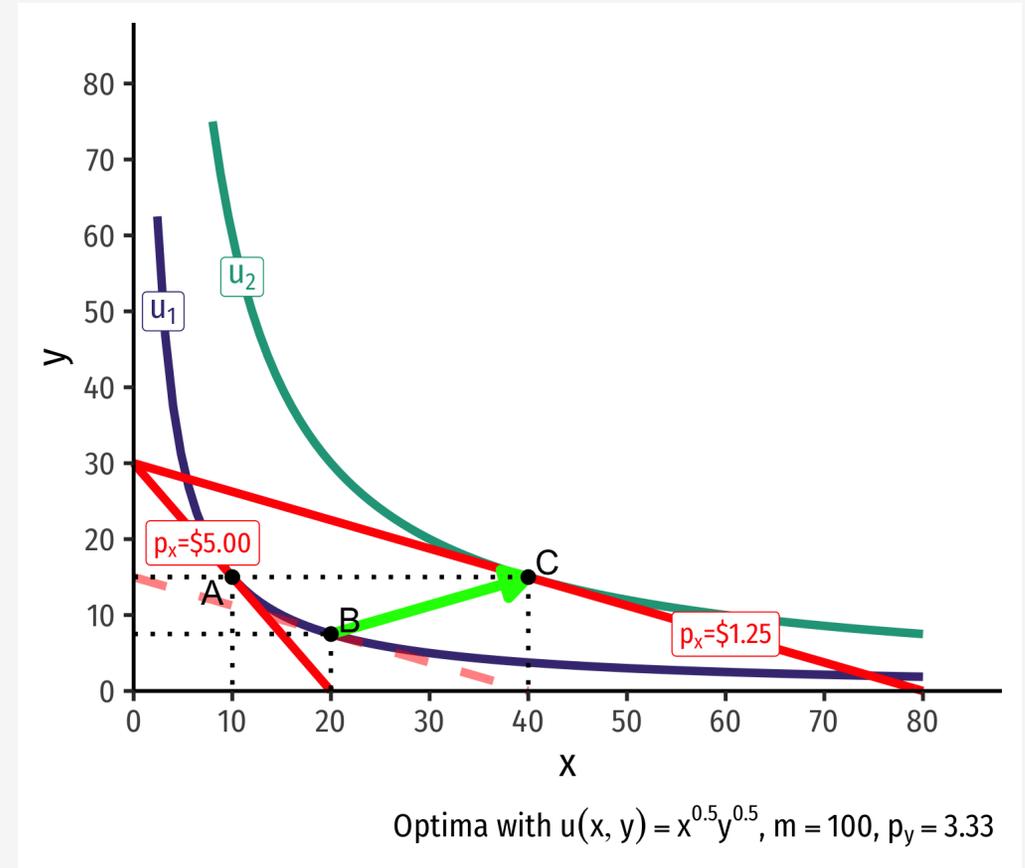
- **(Real) income effect:** change in consumption due to the **change in purchasing power** from the change in price
- $(B \rightarrow C)$ to new budget constraint (can buy more of (x) and/or (y))



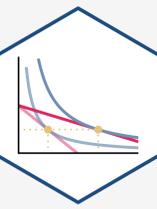
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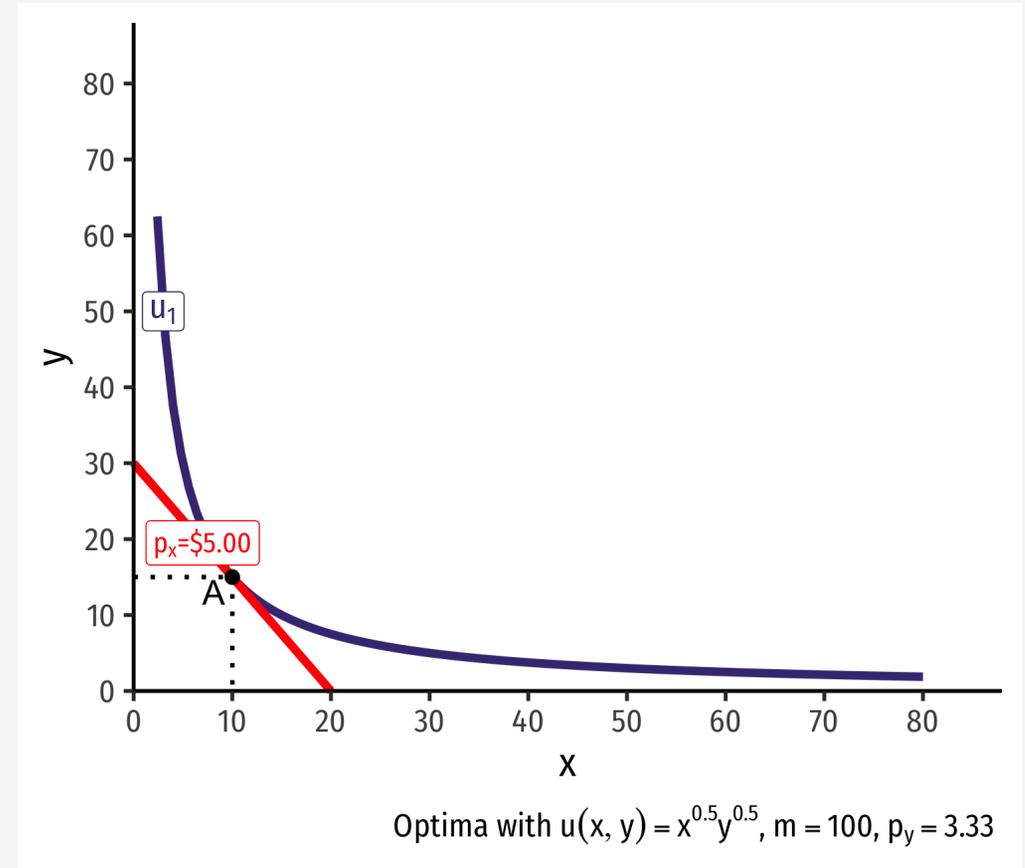
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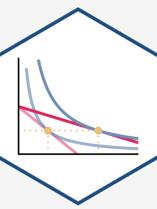
Real Income and Substitution Effects, Graphically IV



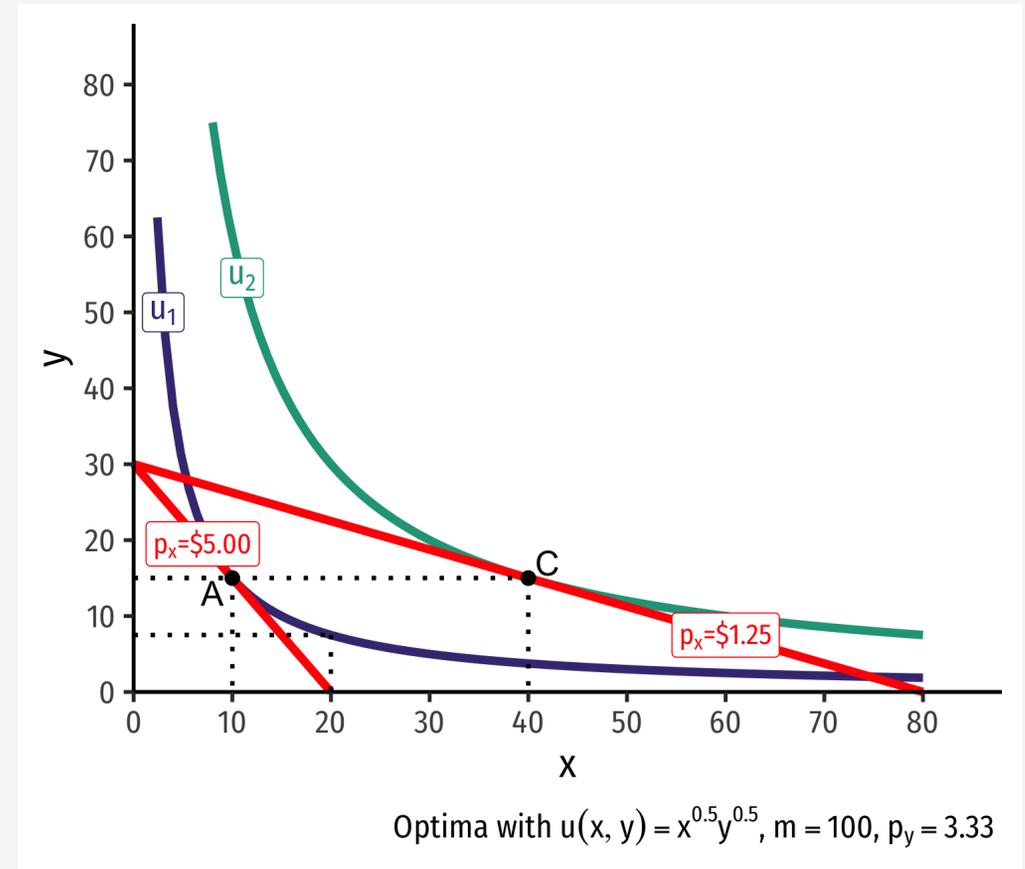
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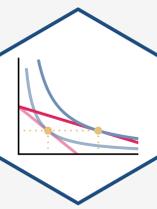
Real Income and Substitution Effects, Graphically IV



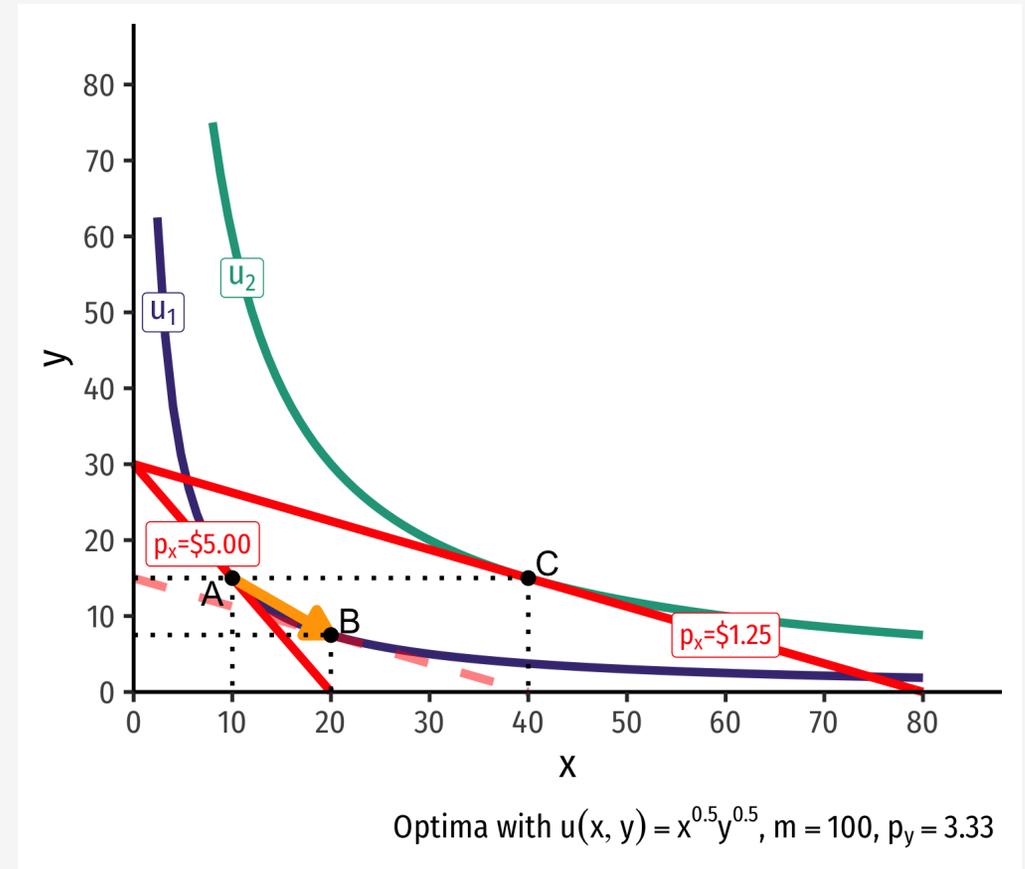
- Original optimal consumption $((A))$
- Price of (x) falls, new optimal consumption at $((C))$



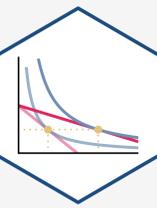
Real Income and Substitution Effects, Graphically IV



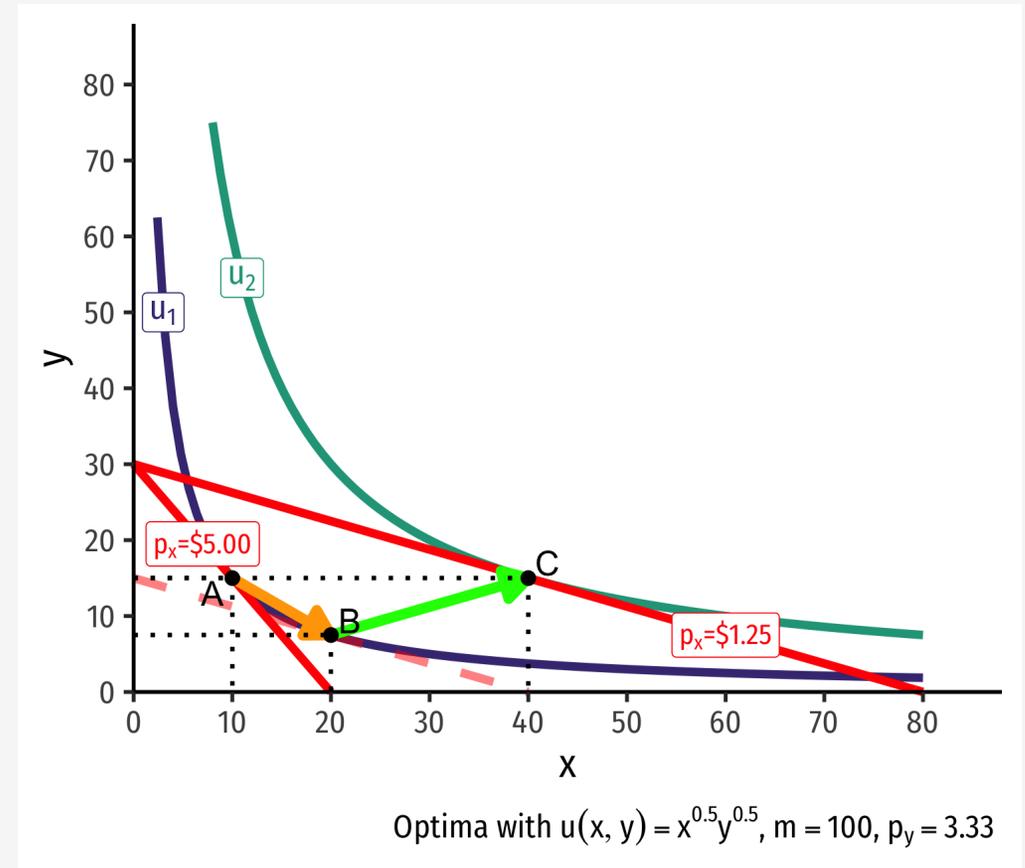
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- **Substitution effect:** $(A \rightarrow B)$ on same I.C. $((\uparrow))$ cheaper (x) and $((\downarrow))$ (y)



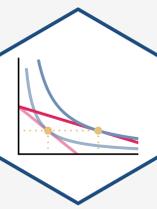
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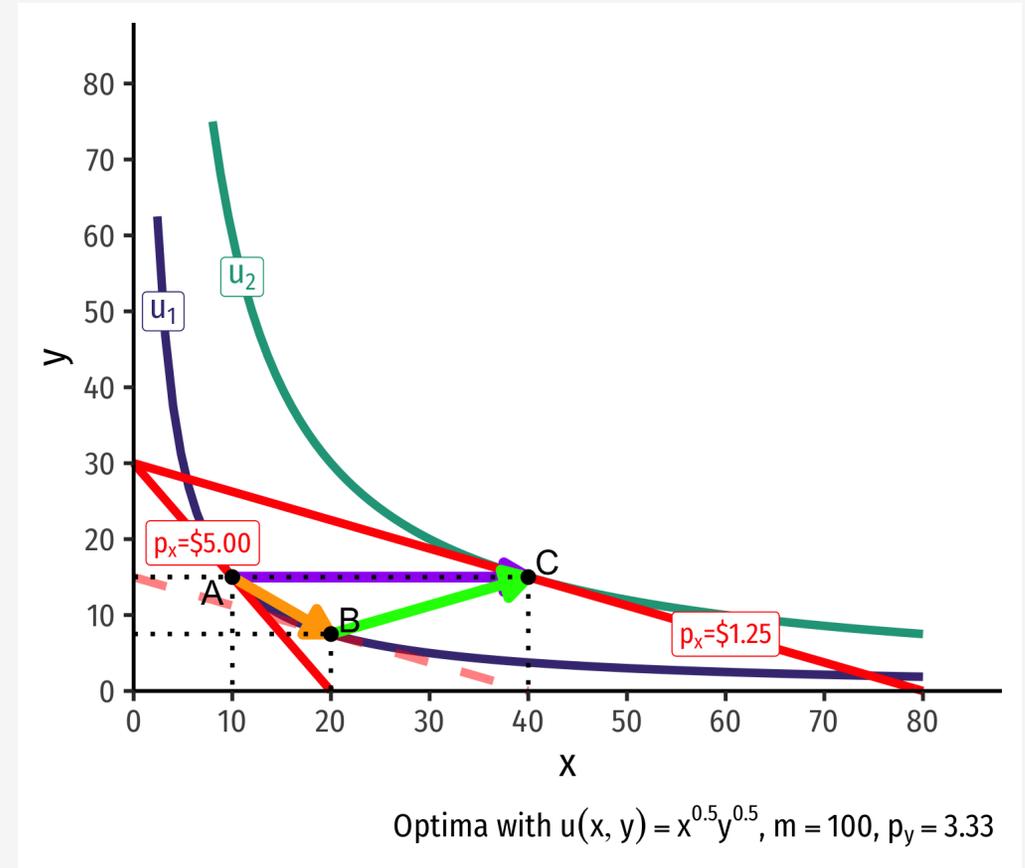
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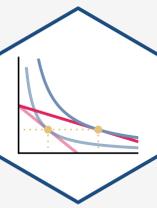
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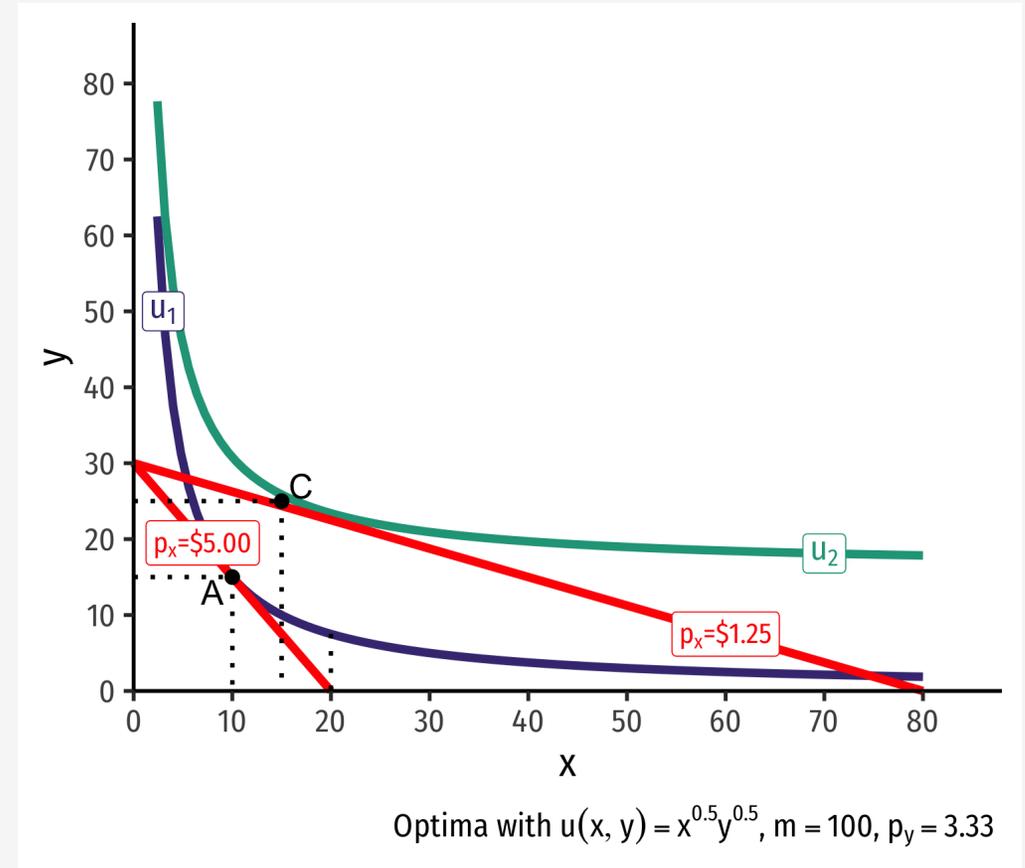
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- **(Real) income effect:** $(B \rightarrow C)$ to new budget constraint (can buy more of (x) and/or (y))
- **(Total) price effect:** $(A \rightarrow C)$



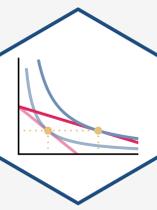
Real Income and Substitution Effects: Inferior Good



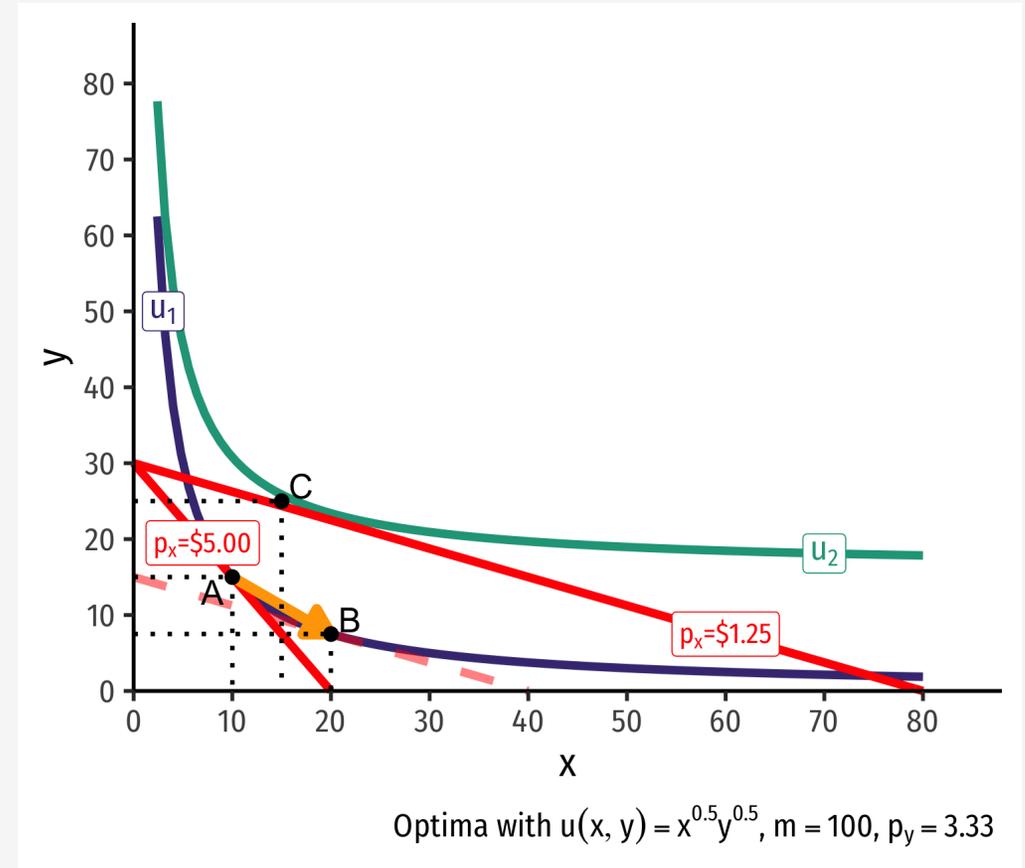
- What about an **inferior** good (Ramen)?



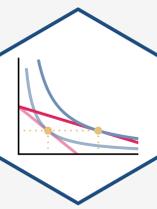
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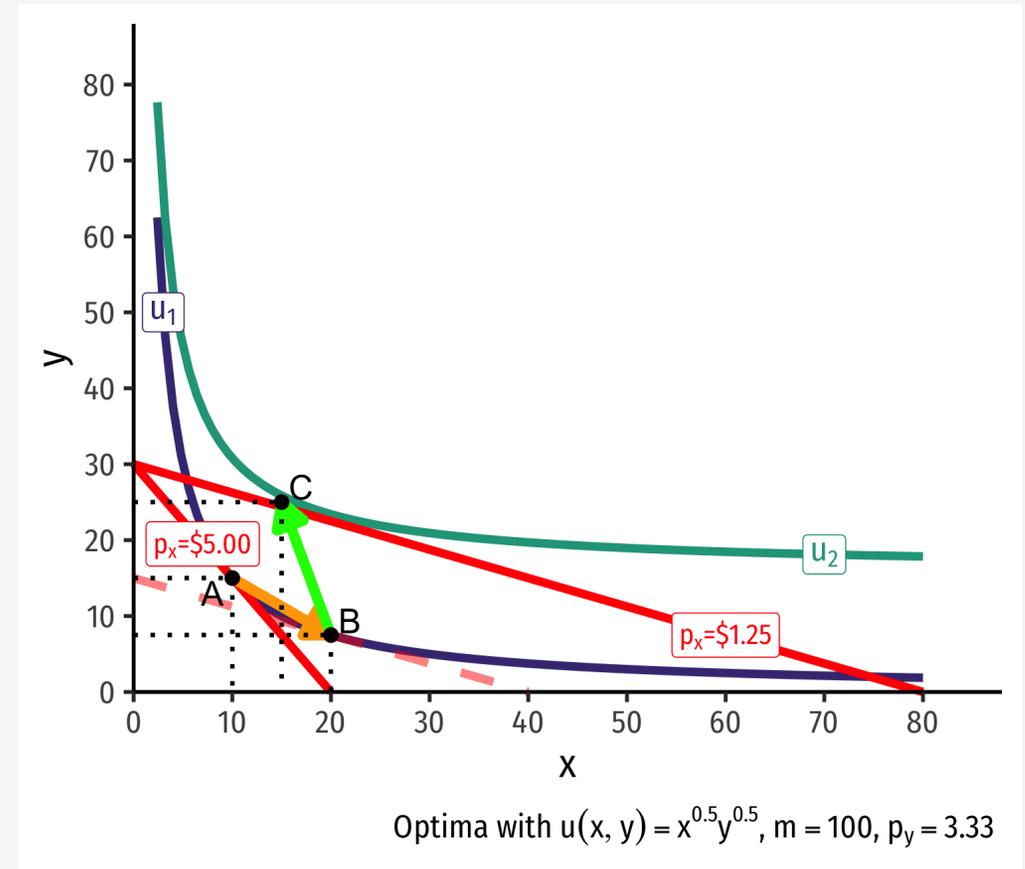
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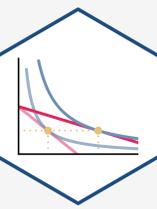
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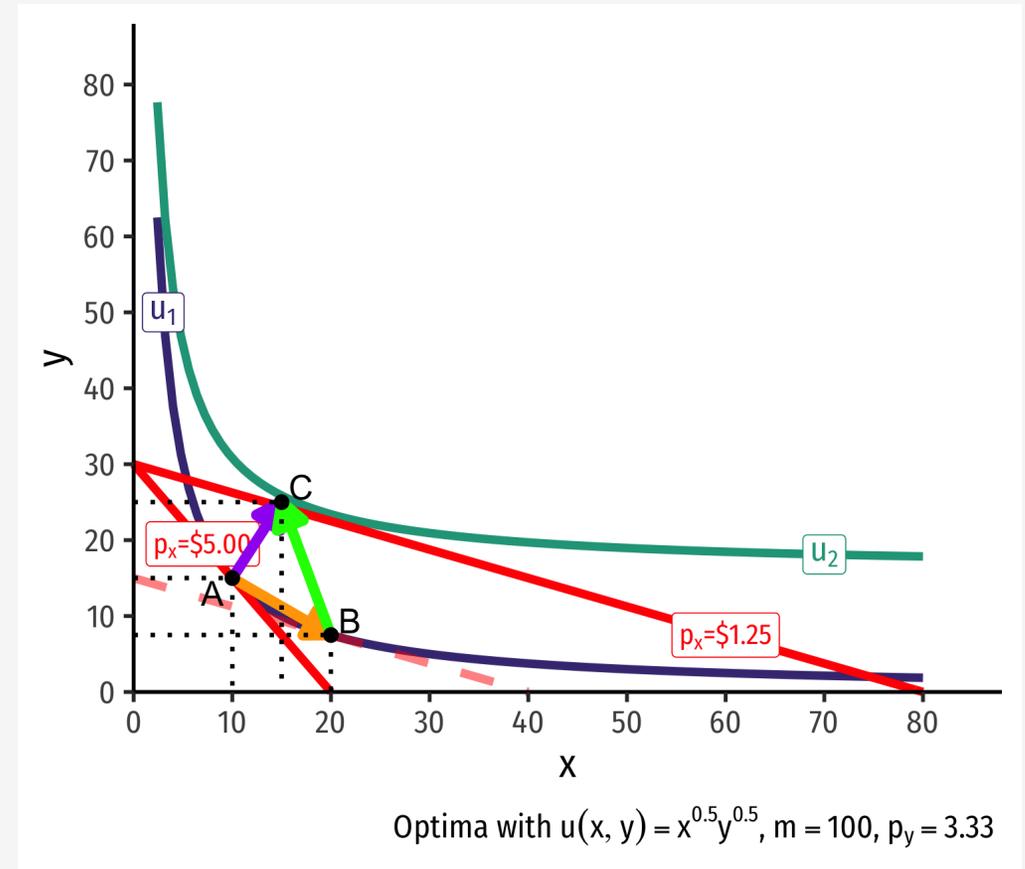
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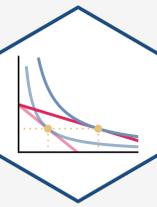
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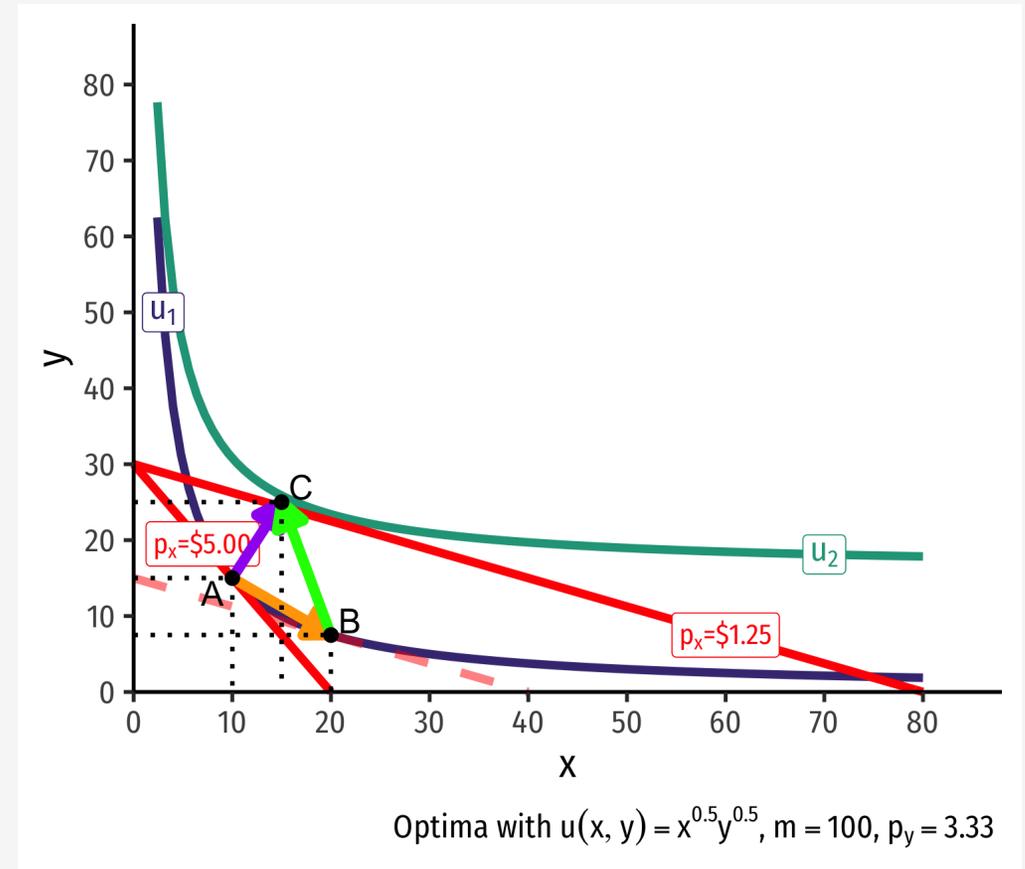
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- **(Total) price effect:** $(A \rightarrow C)$



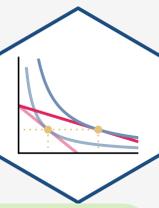
Real Income and Substitution Effects: Inferior Good



- What about an **inferior** good (Ramen)?
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- **(Real) income effect:** $(B \rightarrow C)$ to new budget constraint (can buy more of (x) and/or (y)) **is negative**
- **(Total) price effect:** $(A \rightarrow C)$
- Price effect is *still* an $(\uparrow x)$ from a $(\downarrow p_x)$!

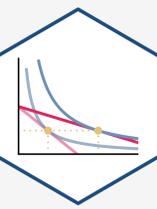


Violating the Law of Demand



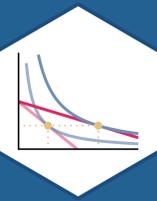
Example: What would it take to violate the law of demand?

Recap: Real Income and Substitution Effects



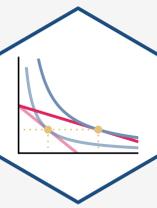
Price Effect \searrow (= \searrow) **Real income effect** \searrow (\rightarrow) **Substitution Effect**

- **Substitution effect**: is always in the direction of the cheaper good
- **Real Income effect**: can be positive (normal) or negative (inferior)
- **Law of Demand**/Demand curves slope downwards (**Price effect**) mostly because of the substitution effect
 - Even (inferior) goods with negative real income effects overpowered by substitution effect
- Exception in the theoretical **Giffen good**: negative R.I.E. \searrow (\rightarrow) S.E.
 - An upward sloping demand curve!



From Optimal Consumption Points to Demand

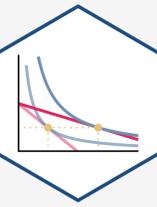
Demand Schedule



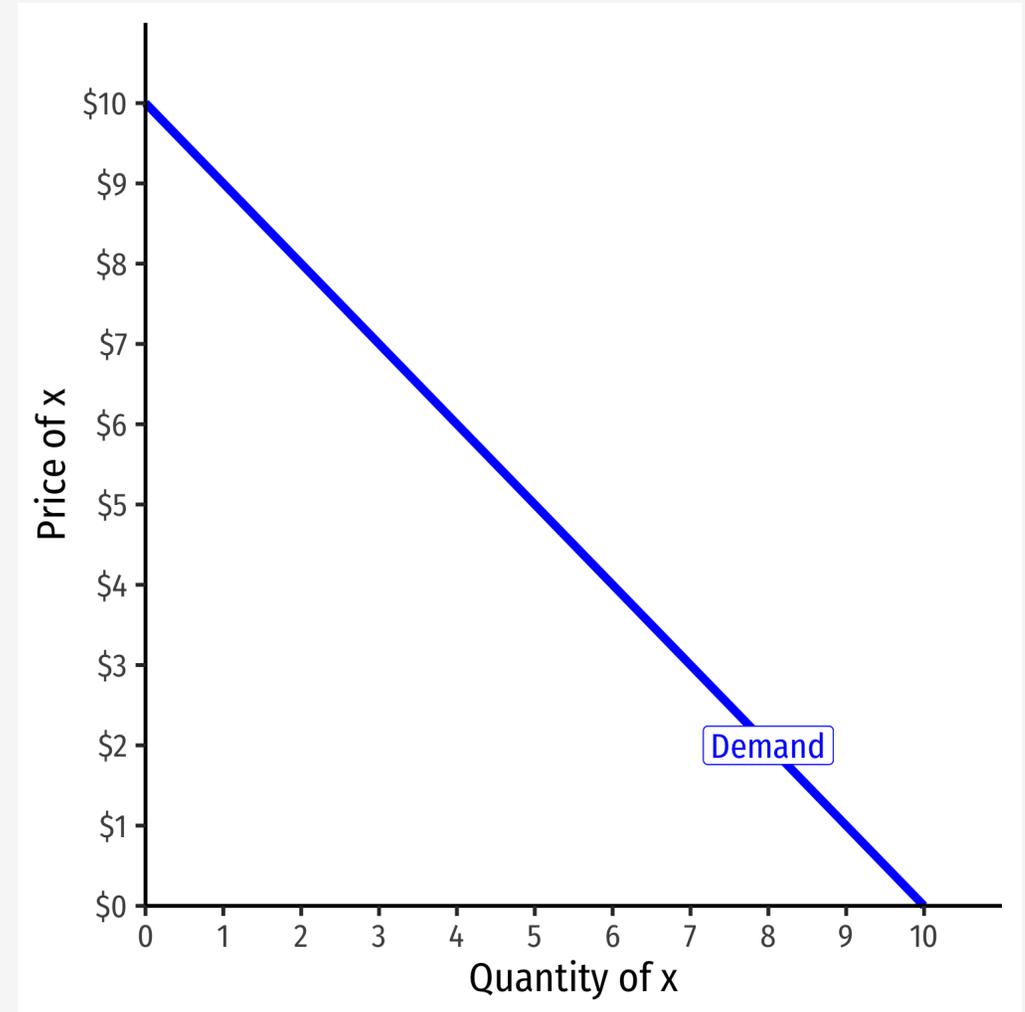
- **Demand schedule** expresses the quantity of good a person would be willing to buy (q_D) at any given price (p_x)
- Note: **each of these is a consumer's optimum at a given price!**

price	quantity
10	0
9	1
8	2
7	3
6	4
5	5
4	6
3	7
2	8
1	9

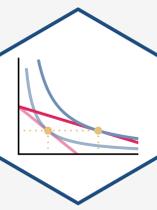
Demand Curve



- **Demand curve** graphically represents the demand schedule
- Also measures a person's **maximum willingness to pay (WTP)** for a given quantity
- Law of Demand (price effect) \implies Demand curves always slope downwards



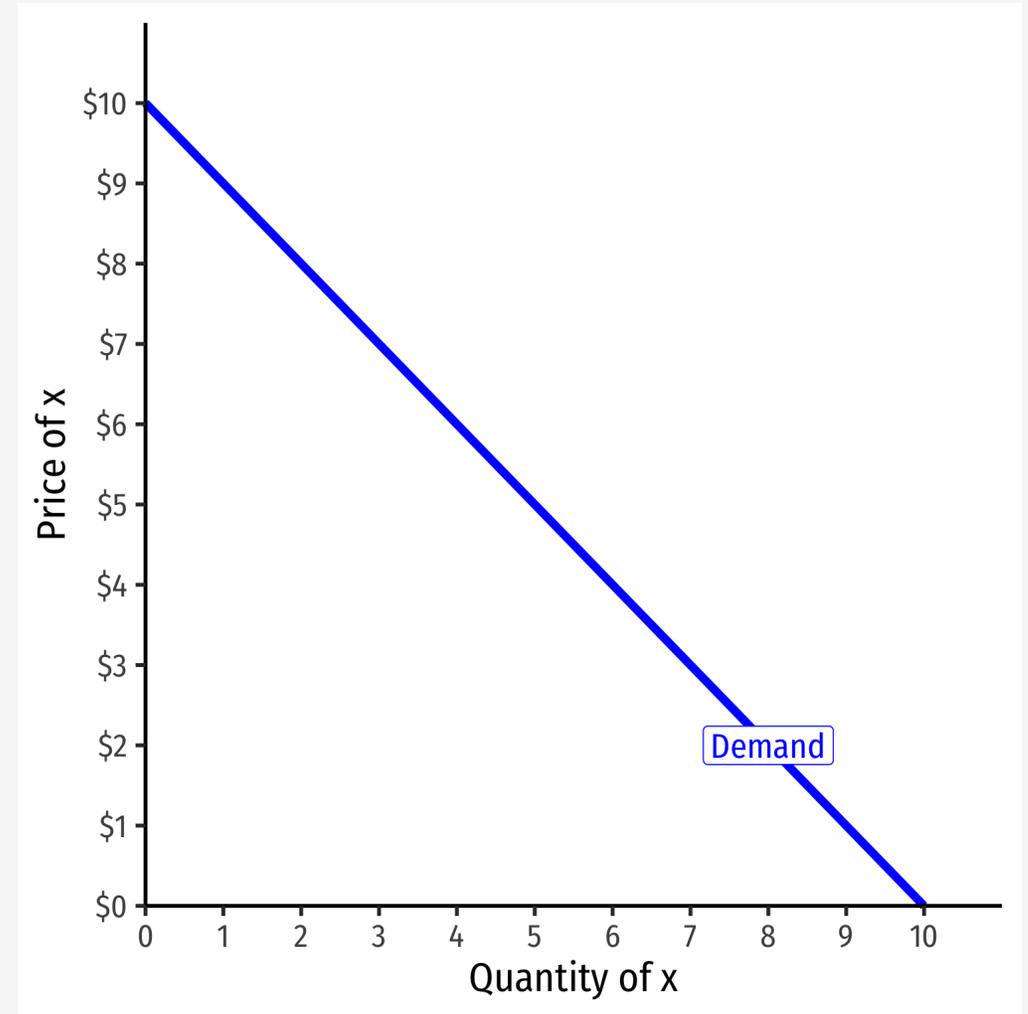
Demand Function



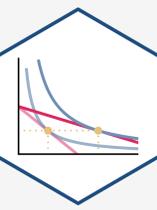
- **Demand function** relates quantity to price

Example: $q=10-p$

- Not graphable (wrong axes)!



Inverse Demand Function

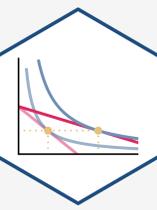


- **Inverse demand function** relates price to quantity
 - Take demand function and solve for p

Example: $p=10-q$

- Graphable (price on vertical axis)!

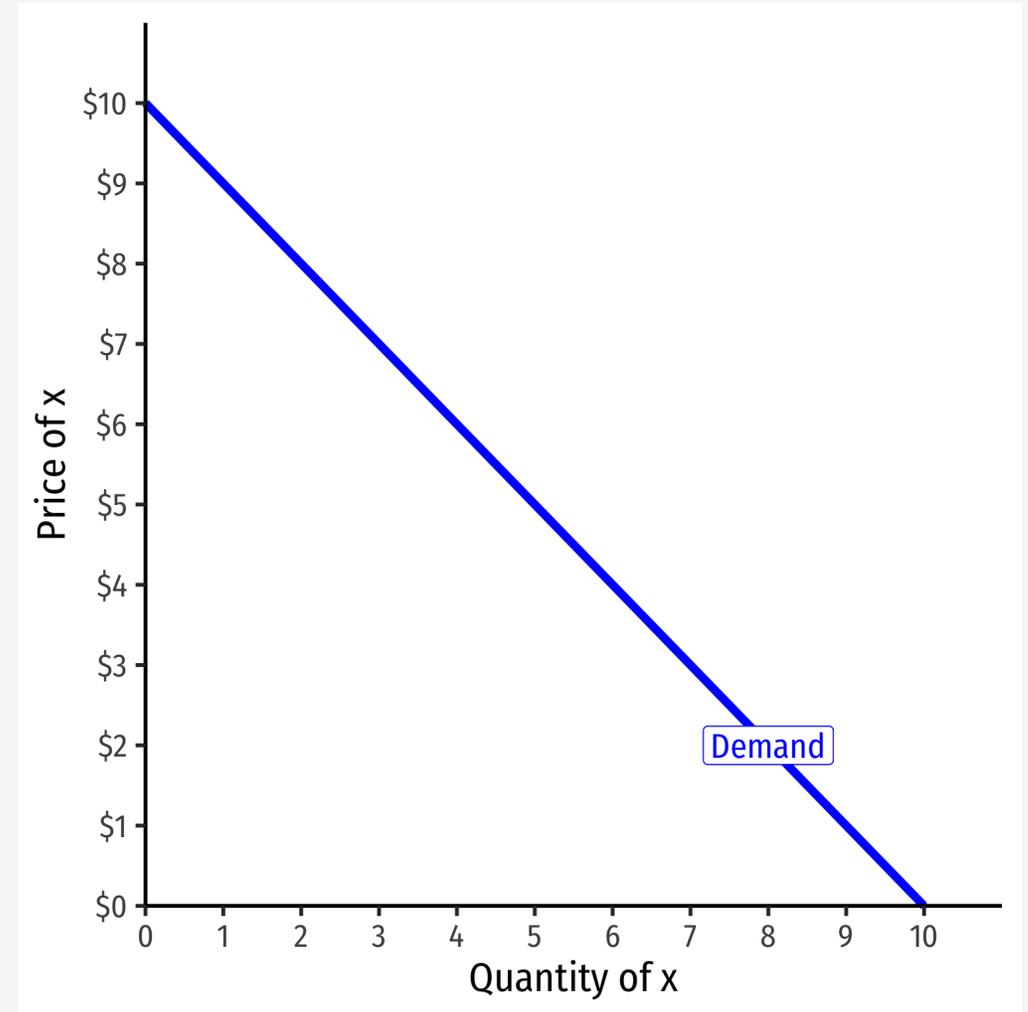
Inverse Demand Function



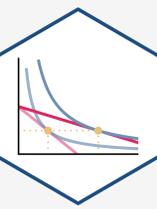
- **Inverse demand function** relates price to quantity
 - Take demand function and solve for p

Example: $p=10-q$

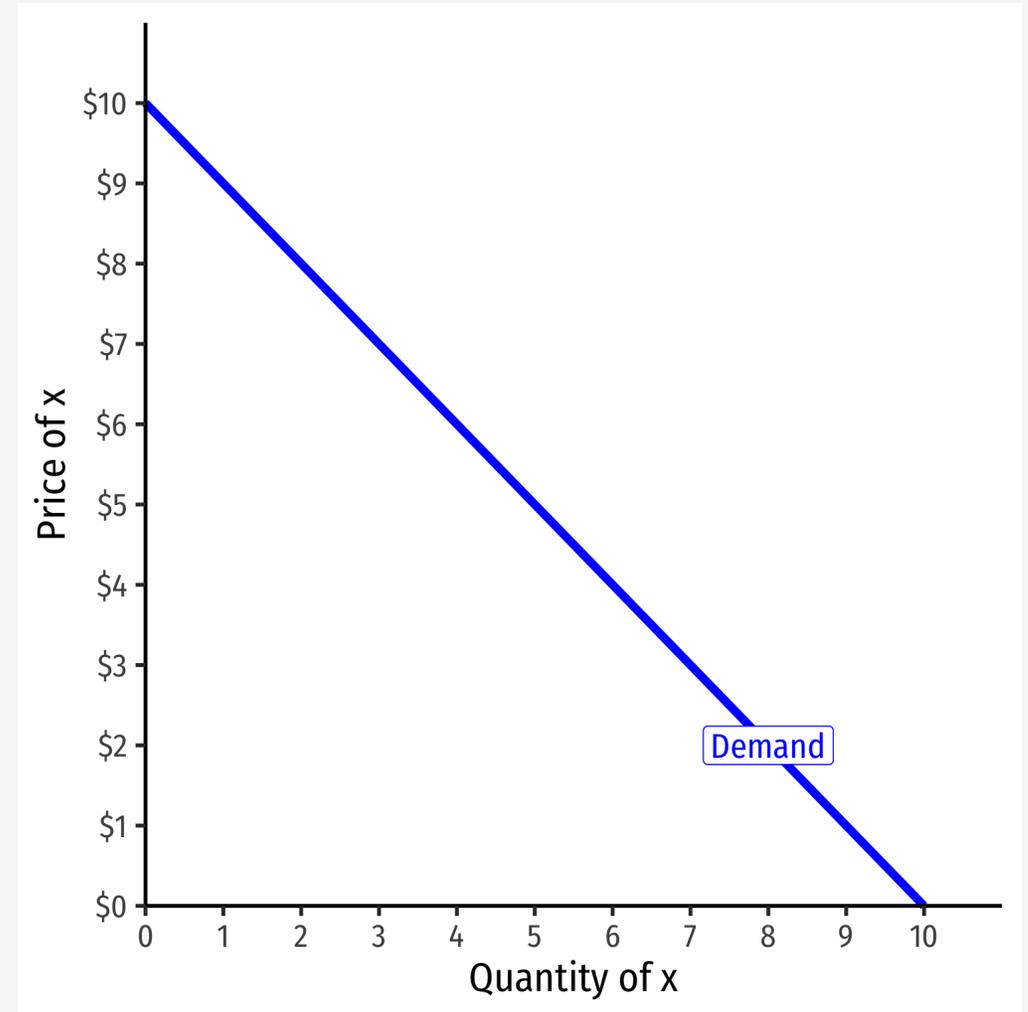
- Vertical intercept ("**Choke price**"): price where $(q_D=0)$ (\$10), just high enough to discourage *any* purchases



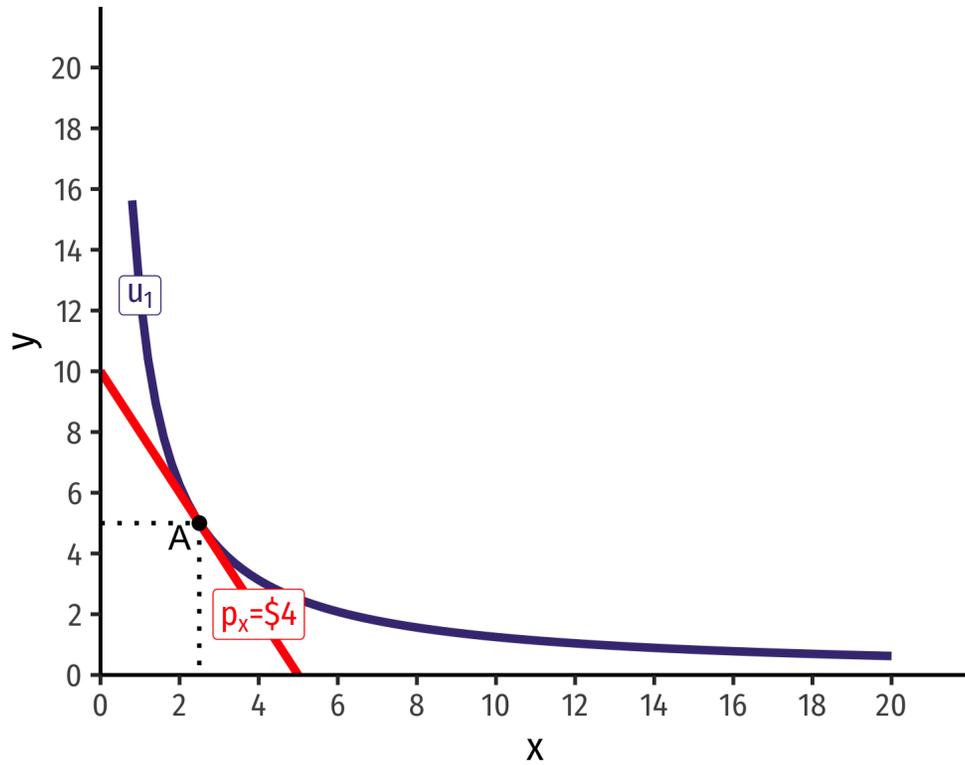
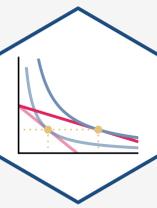
Inverse Demand Function



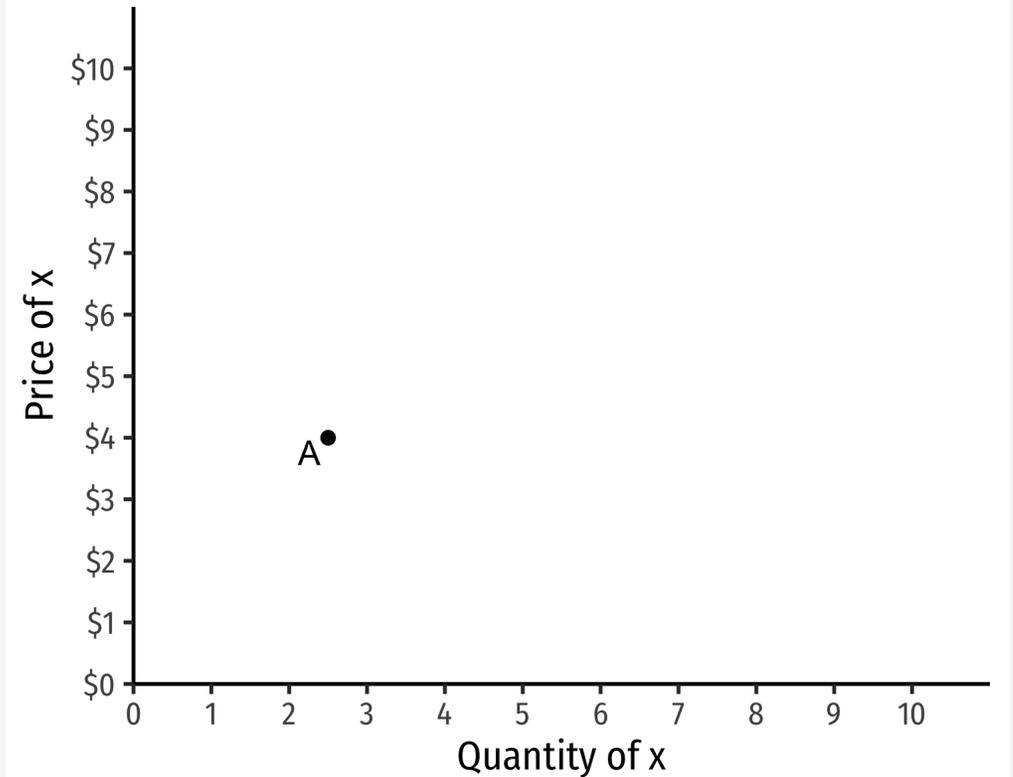
- Read two ways:
- Horizontally: at any given price, how many units person wants to buy
- Vertically: at any given quantity, the **maximum willingness to pay (WTP)** for that quantity
 - This way will be very useful later



Deriving a Demand Curve Graphically

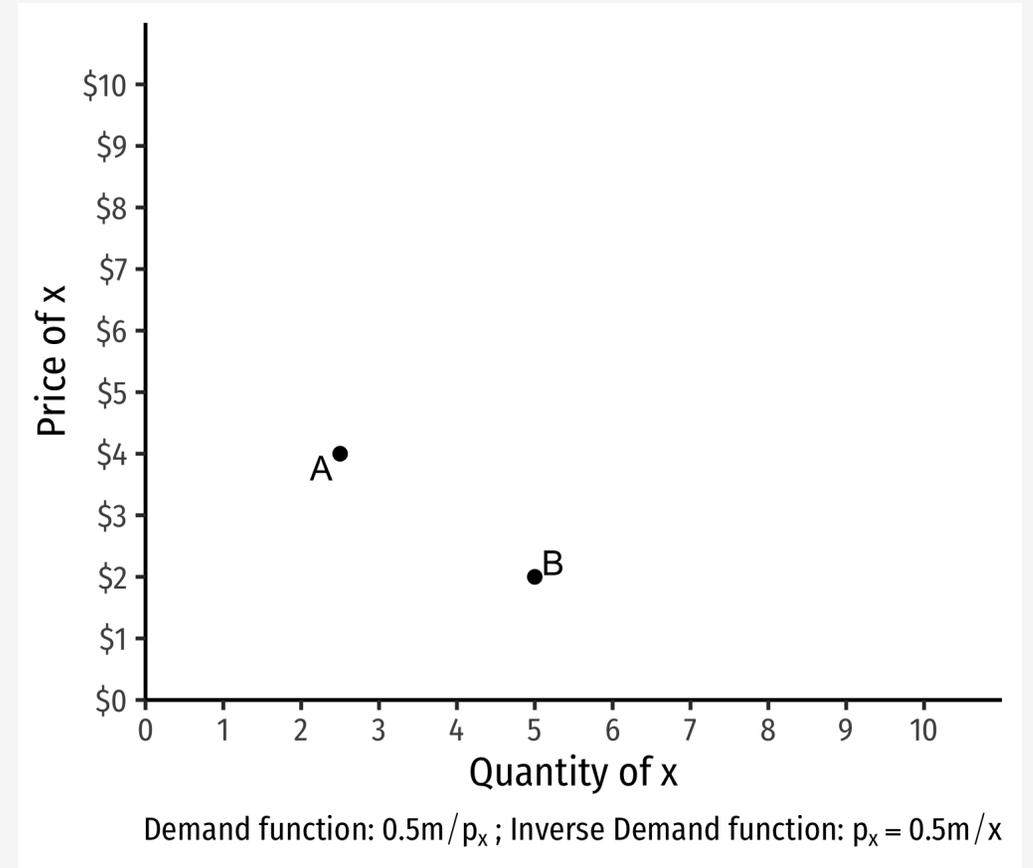
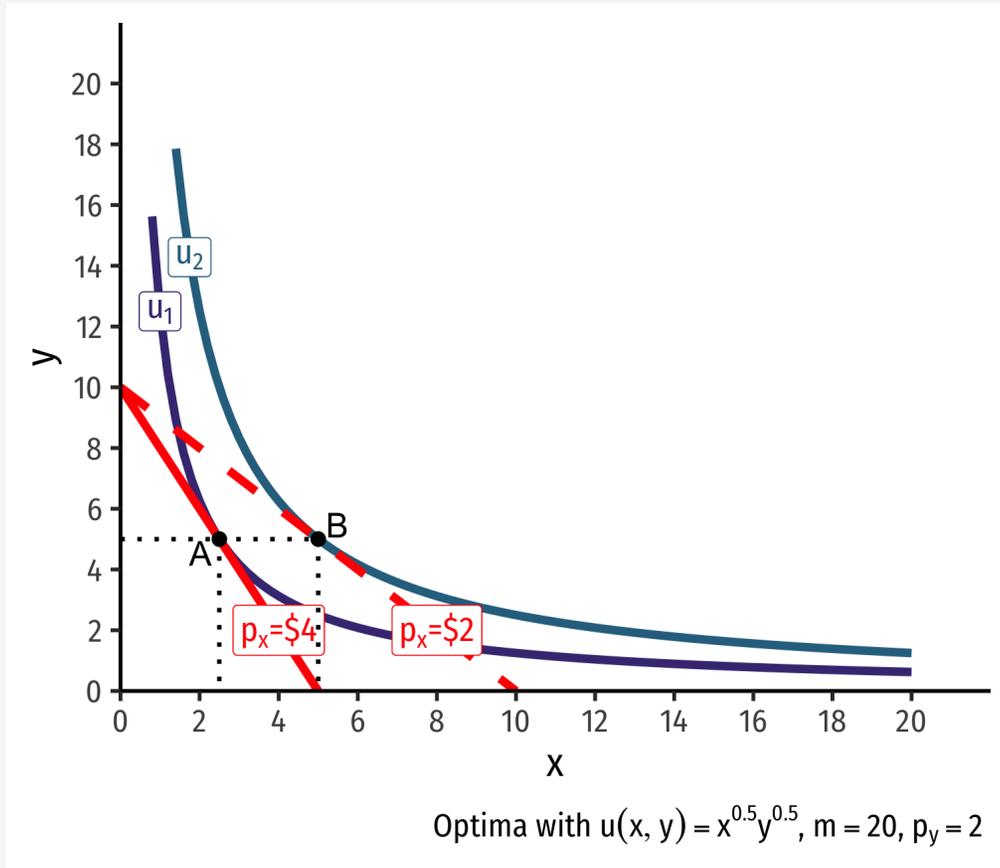
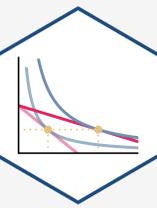


Optima with $u(x, y) = x^{0.5}y^{0.5}$, $m = 20$, $p_y = 2$

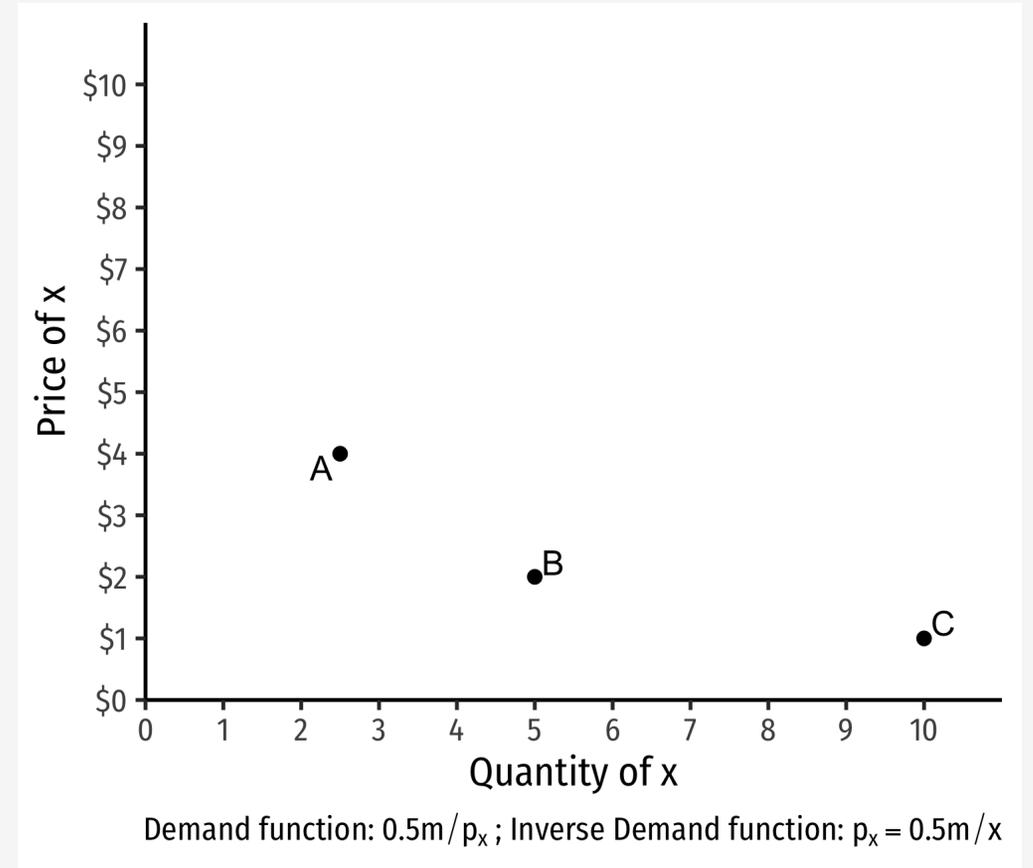
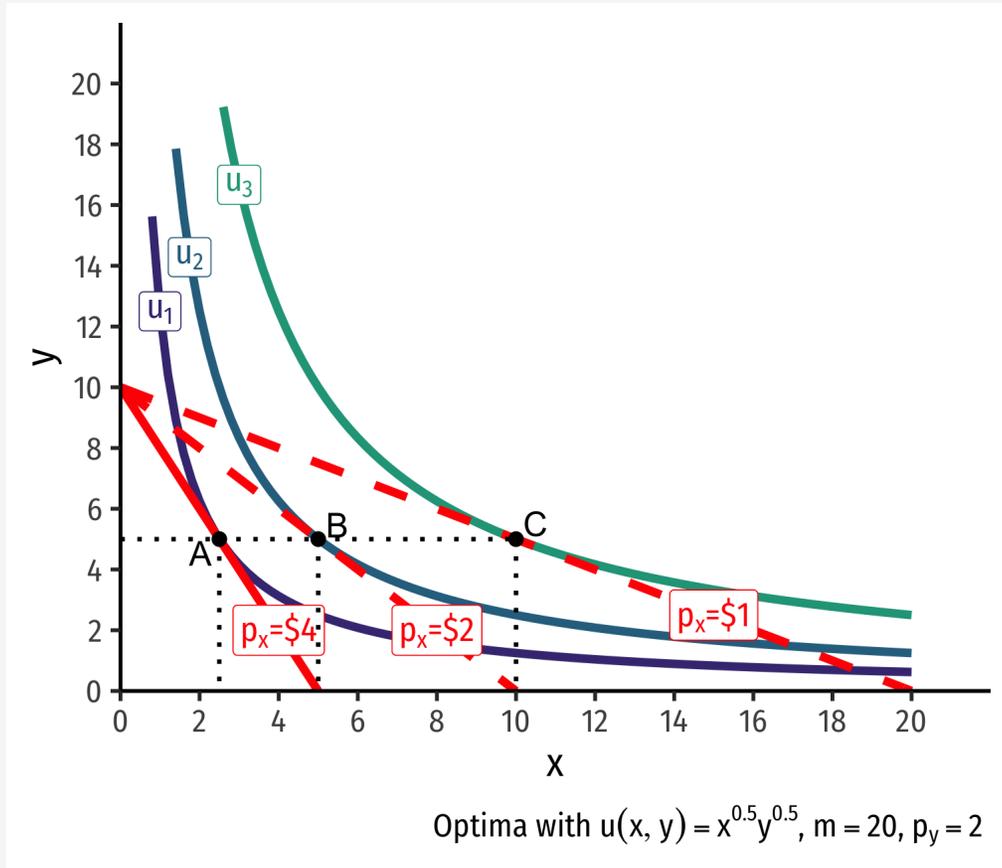
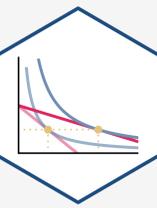


Demand function: $0.5m/p_x$; Inverse Demand function: $p_x = 0.5m/x$

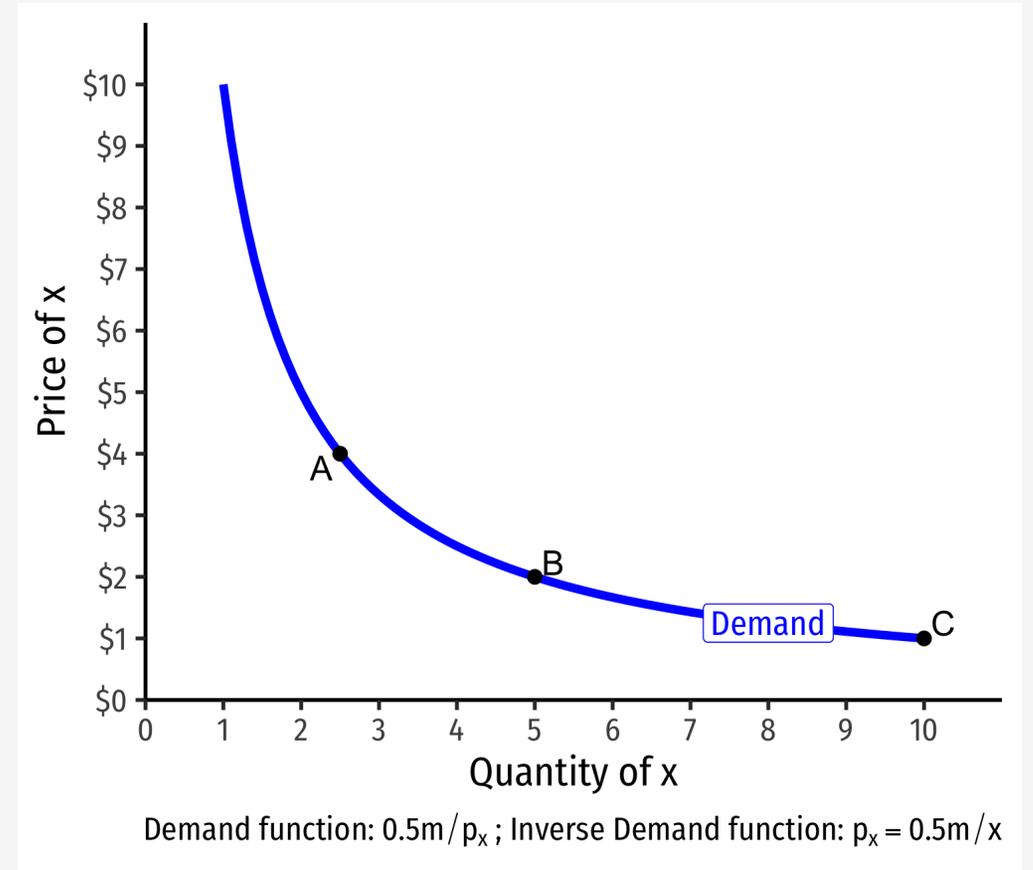
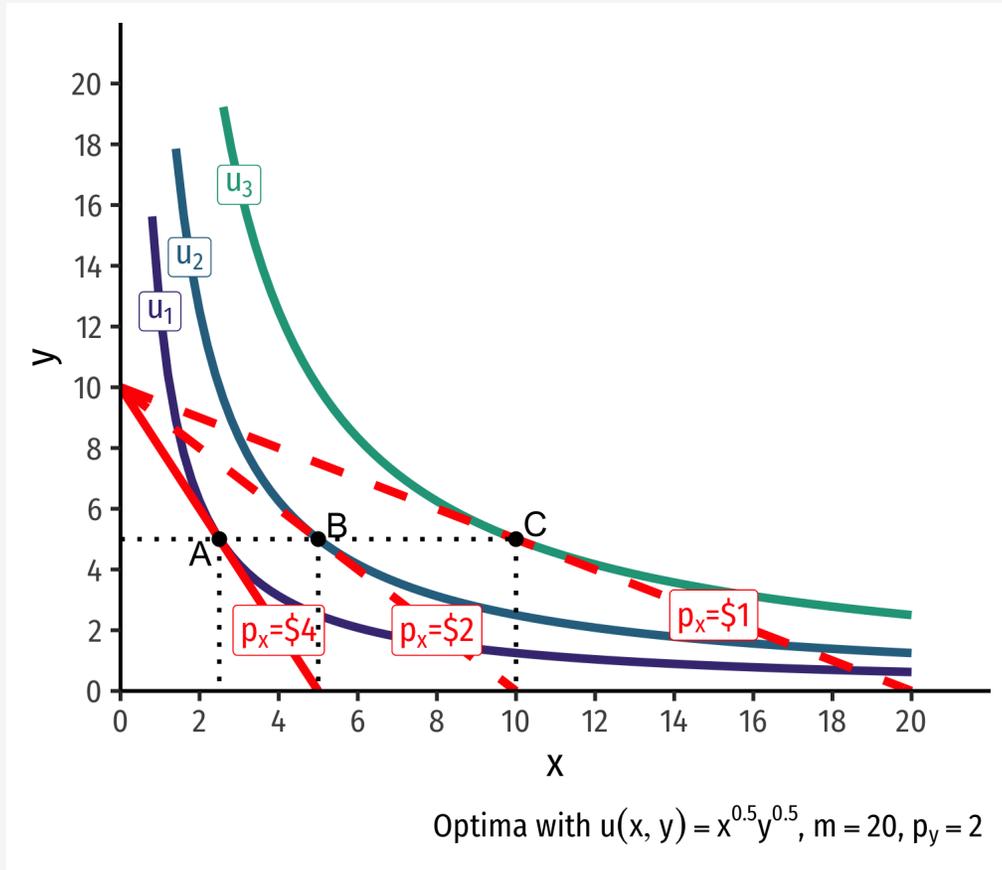
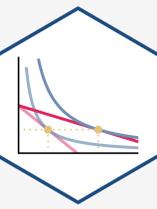
Deriving a Demand Curve Graphically



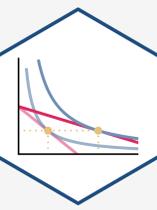
Deriving a Demand Curve Graphically



Deriving a Demand Curve Graphically

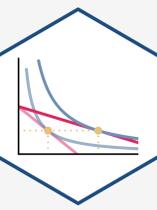


Deriving a Demand *Function* I



- I will always give you a (linear) demand function
- Today's class notes page shows how you can derive actual demand functions from utility functions

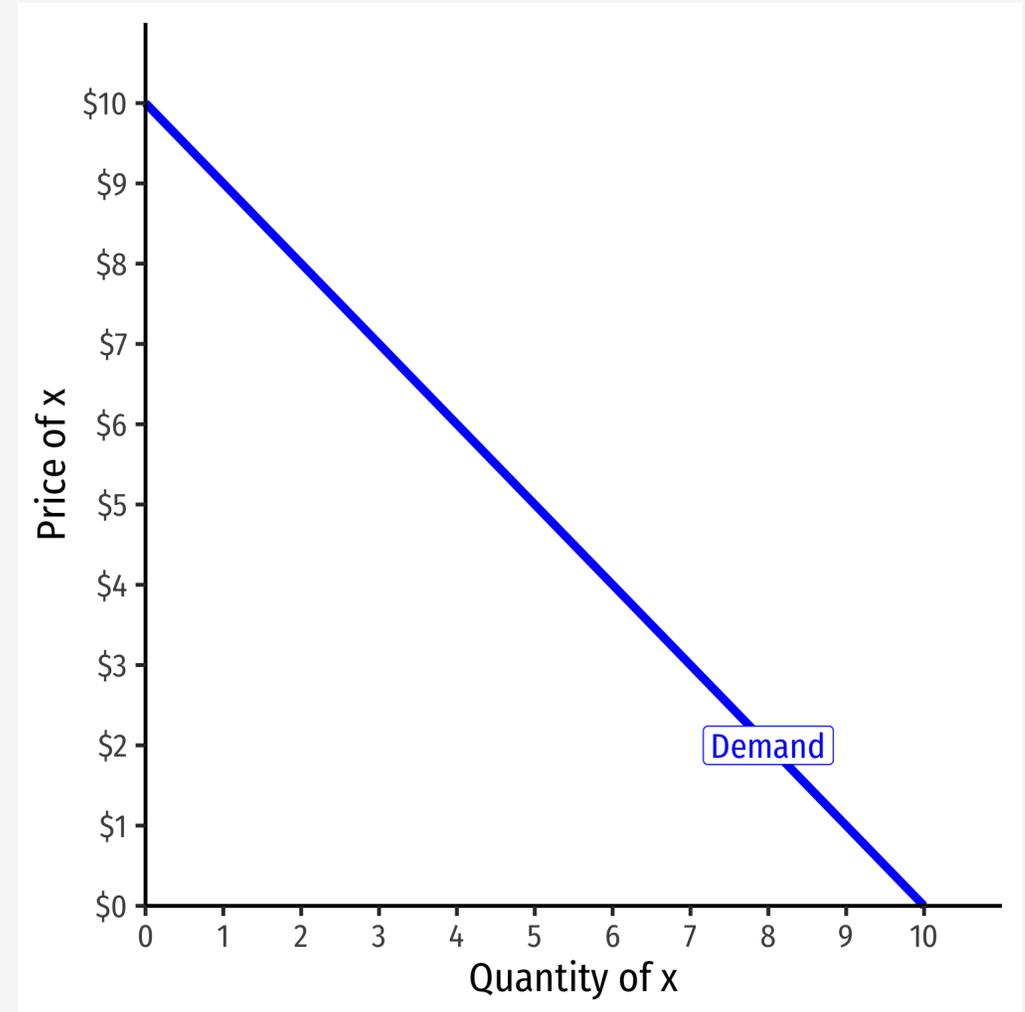
Shifts in Demand I



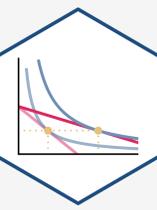
- Note a simple (inverse) demand function only relates (own) **price** and **quantity**

Example: $(q=10-p)$ or $(p=10-q)$

- What about all the other "**determinants of demand**" like income and other prices?
- They are captured in the vertical intercept (choke price)!



Shifts in Demand II



- A change in one of the "**determinants of demand**" will **shift** demand curve!
 - Change in **income** (m)
 - Change in **price of other goods** (p_y) (substitutes or complements)
 - Change in **preferences** or **expectations** about good (x)
 - Change in **number of buyers**
- Shows up in (inverse) demand function by a **change in intercept (choke price)**!
- See my [Visualizing Demand Shifters](#)

