2.5 — Short Run Profit Maximization

ECON 306 · Microeconomic Analysis · Fall 2020

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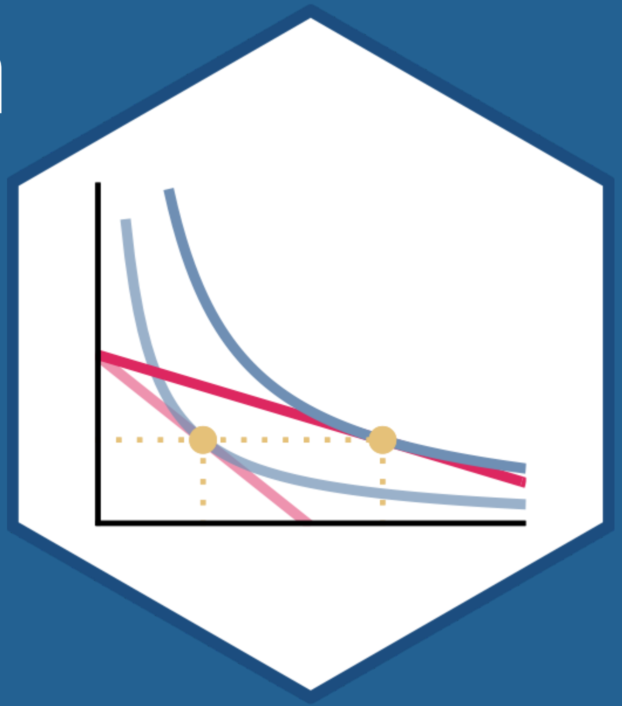
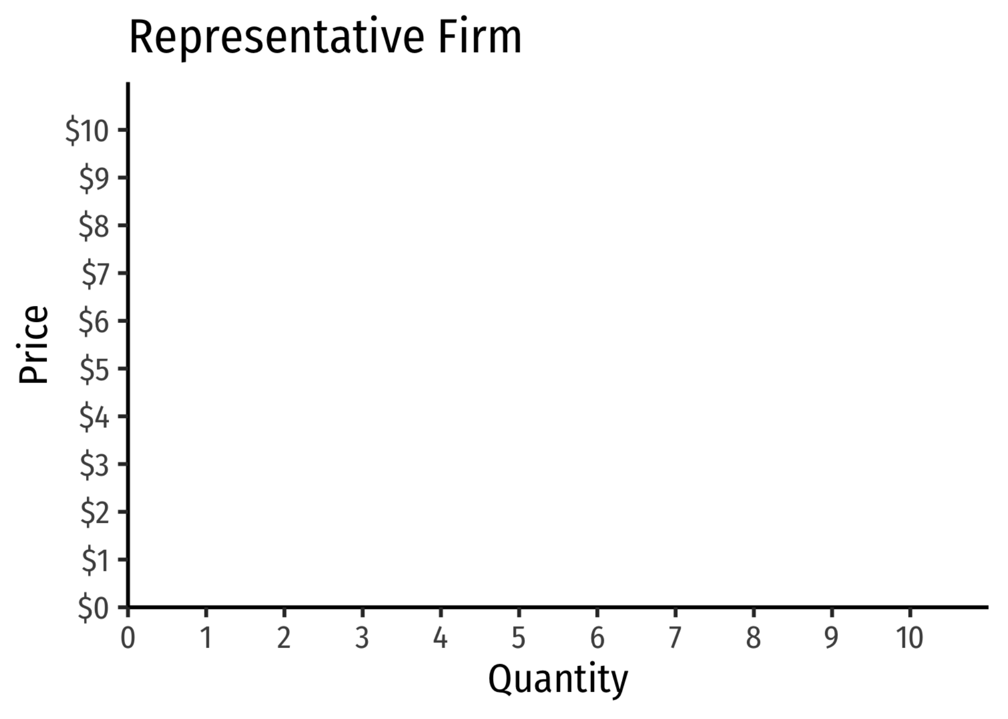
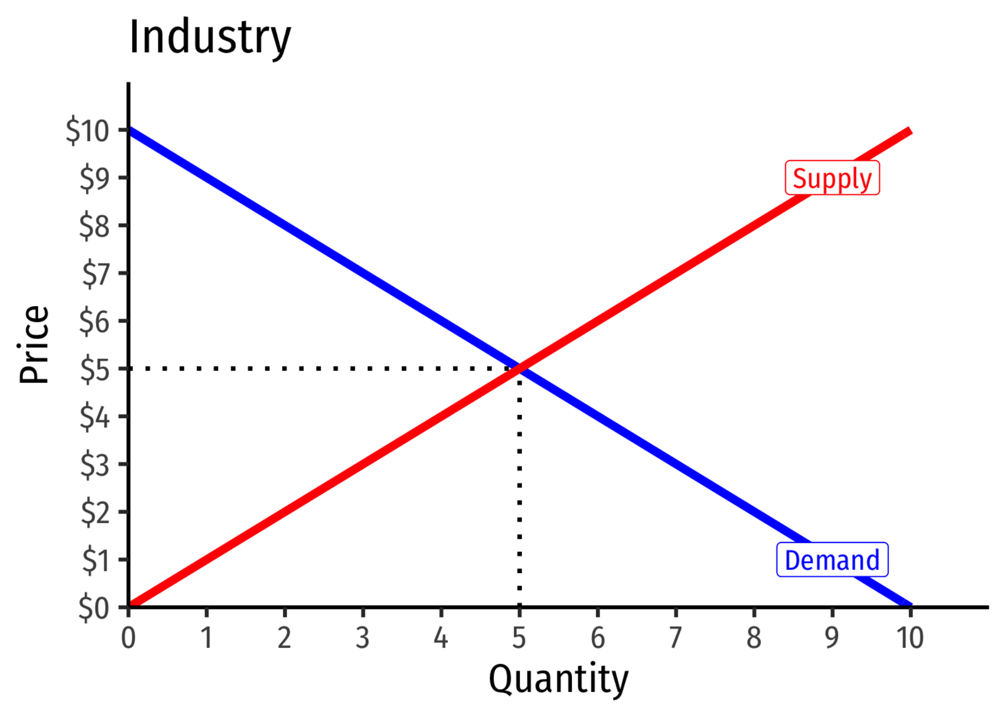


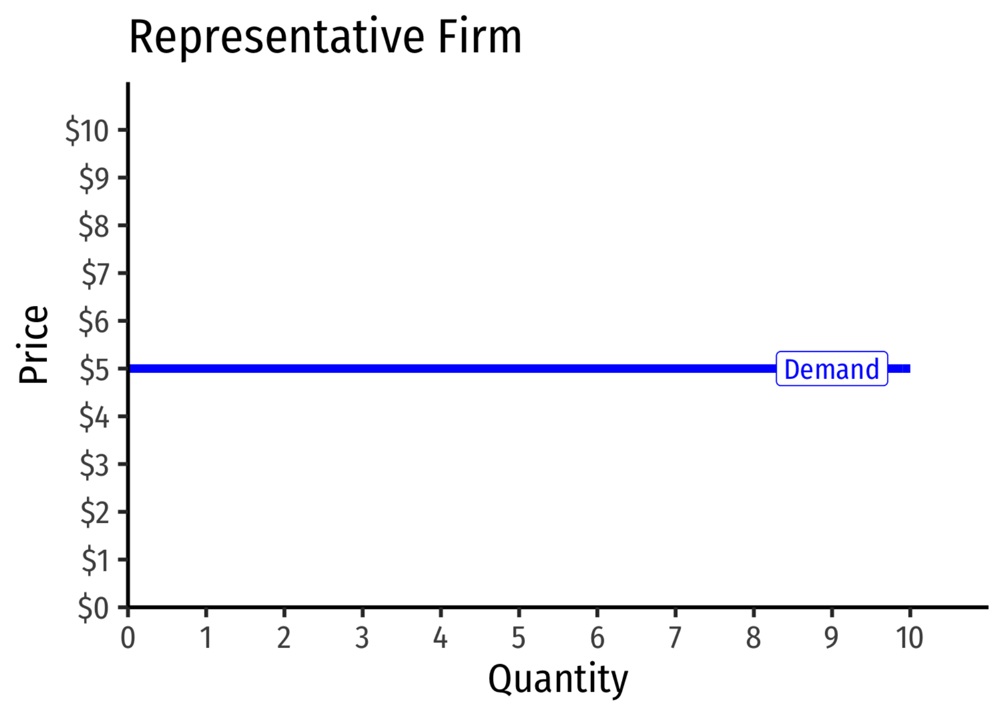
Revenues

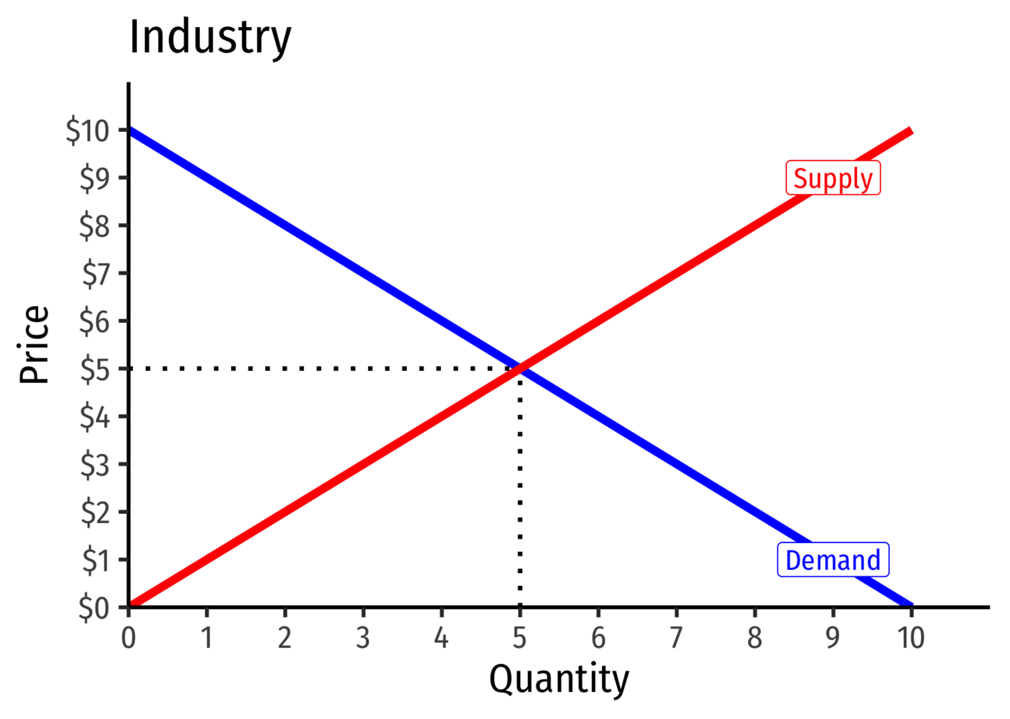
Revenues for Firms in *Competitive* Industries I





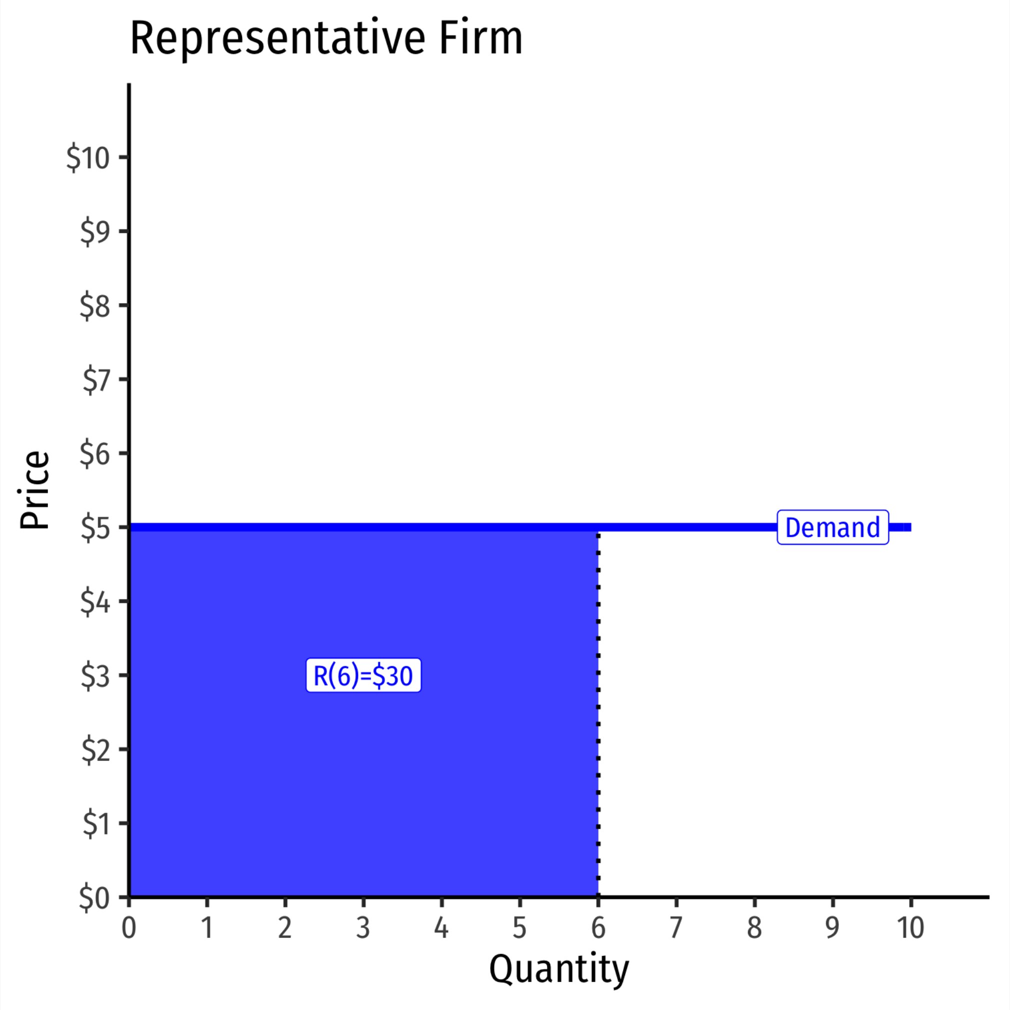
Revenues for Firms in *Competitive* Industries I





- Demand for a firm's product is **perfectly elastic** at the market price
- Where did the supply curve come from? You'll see

Revenues for Firms in *Competitive* Industries II



- **Total Revenue** $R(q) = pq$

Average and Marginal Revenues

- **Average Revenue**: revenue per unit of output

$$AR(q) = \frac{R}{q}$$

  - Is *always* equal to the price! Why?

- **Marginal Revenue**: change in revenues for each additional unit of output sold:

$$MR(q) = \frac{\Delta R(q)}{\Delta q} \approx \frac{R_2 - R_1}{q_2 - q_1}$$

  - Calculus: first derivative of the revenues function
  - For a *competitive* firm, always equal to the price!

Average and Marginal Revenues: Example

Example: A firm sells bushels of wheat in a very competitive market. The current market price is \$10/bushel.

For the 1st bushel sold:

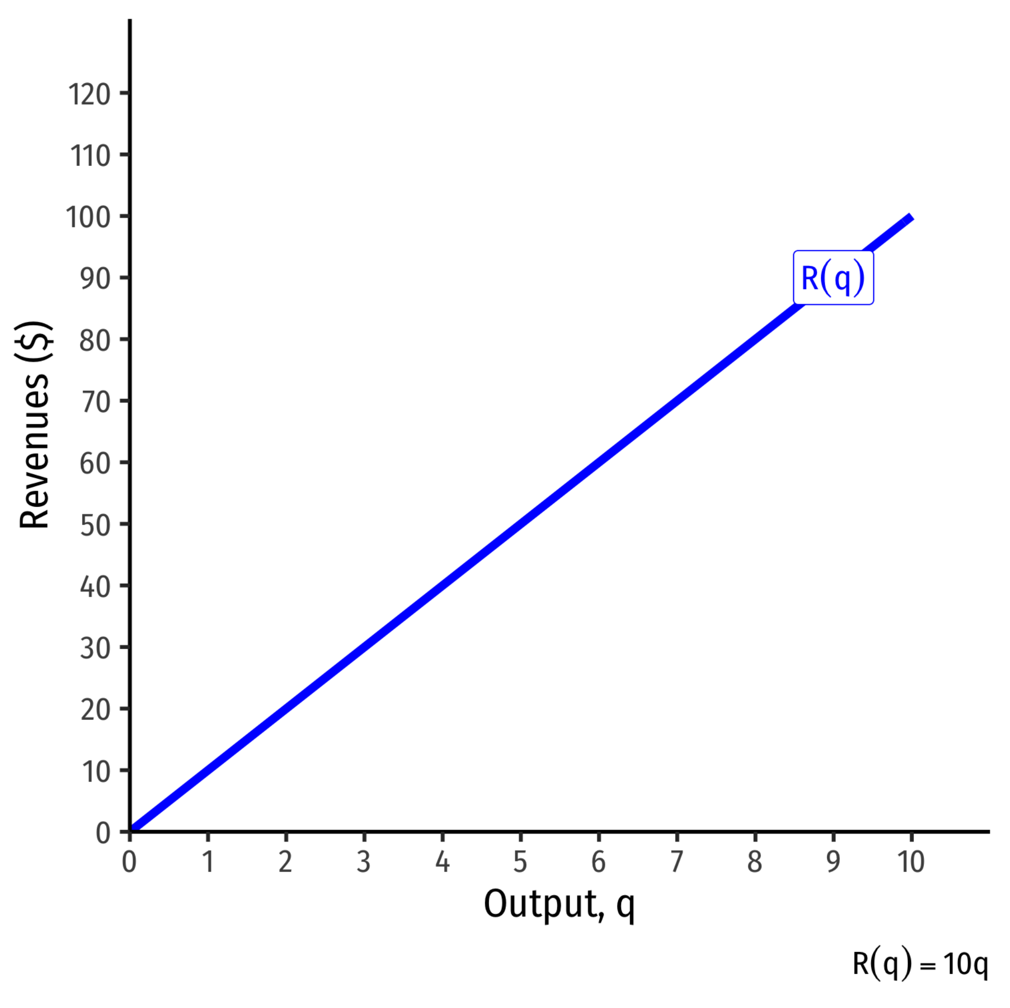
- What is the total revenue?
- What is the average revenue?

For the 2nd bushel sold:

- What is the total revenue?
- What is the average revenue?
- What is the marginal revenue?

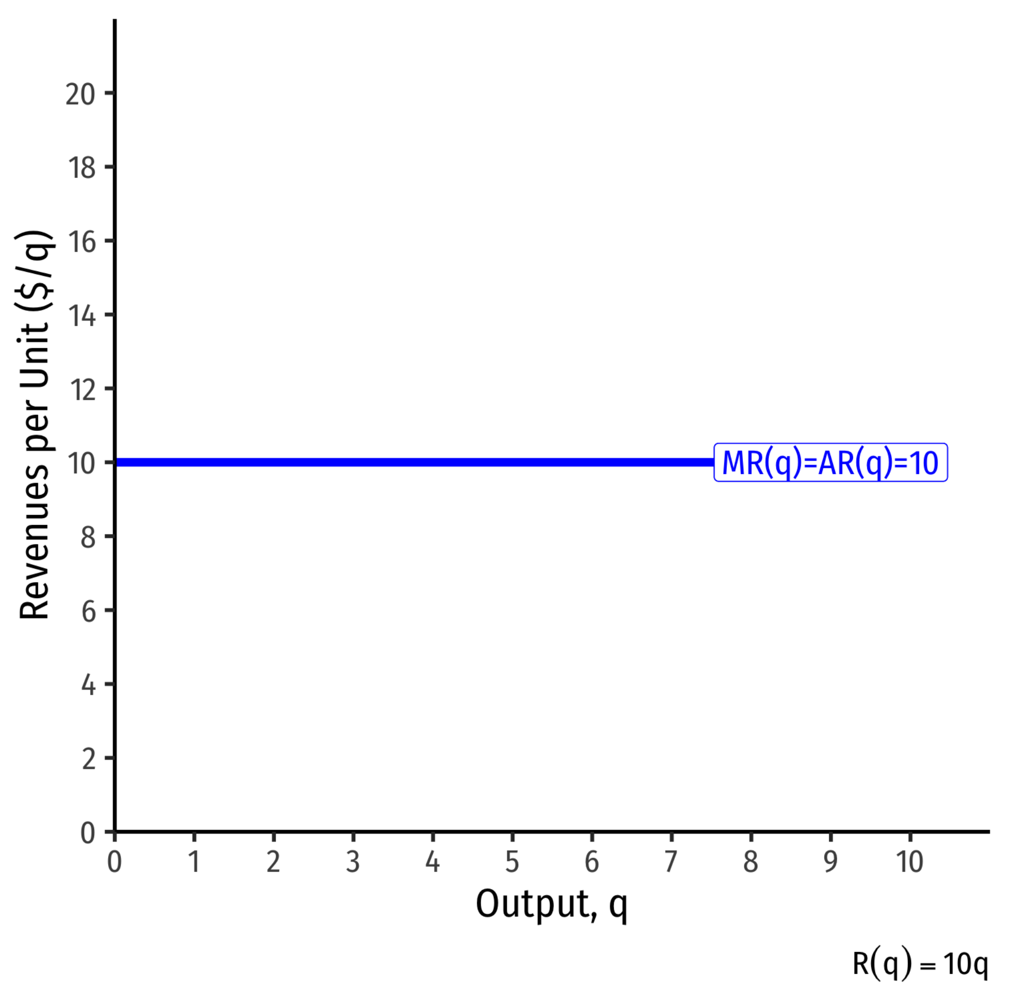
Total Revenue, Example: Visualized

q	$R(q)$
0	0
1	10
2	20
3	30
4	40
5	50
6	60
7	70
8	80
9	90



Average and Marginal Revenue, Example: Visualized

q	$R(q)$	$AR(q)$	$MR(q)$
0	0	–	–
1	10	10	10
2	20	10	10
3	30	10	10
4	40	10	10
5	50	10	10
6	60	10	10
7	70	10	10
8	80	10	10
9	90	10	10



Recall: The Firm's Two Problems

- 1^{st} Stage: **firm's profit maximization problem**:

1. **Choose:** **< output >**

2. **In order to maximize:** **< profits >**

- We'll cover this later...first we'll explore:

- 2^{nd} Stage: **firm's cost minimization problem**:

1. **Choose:** **< inputs >**

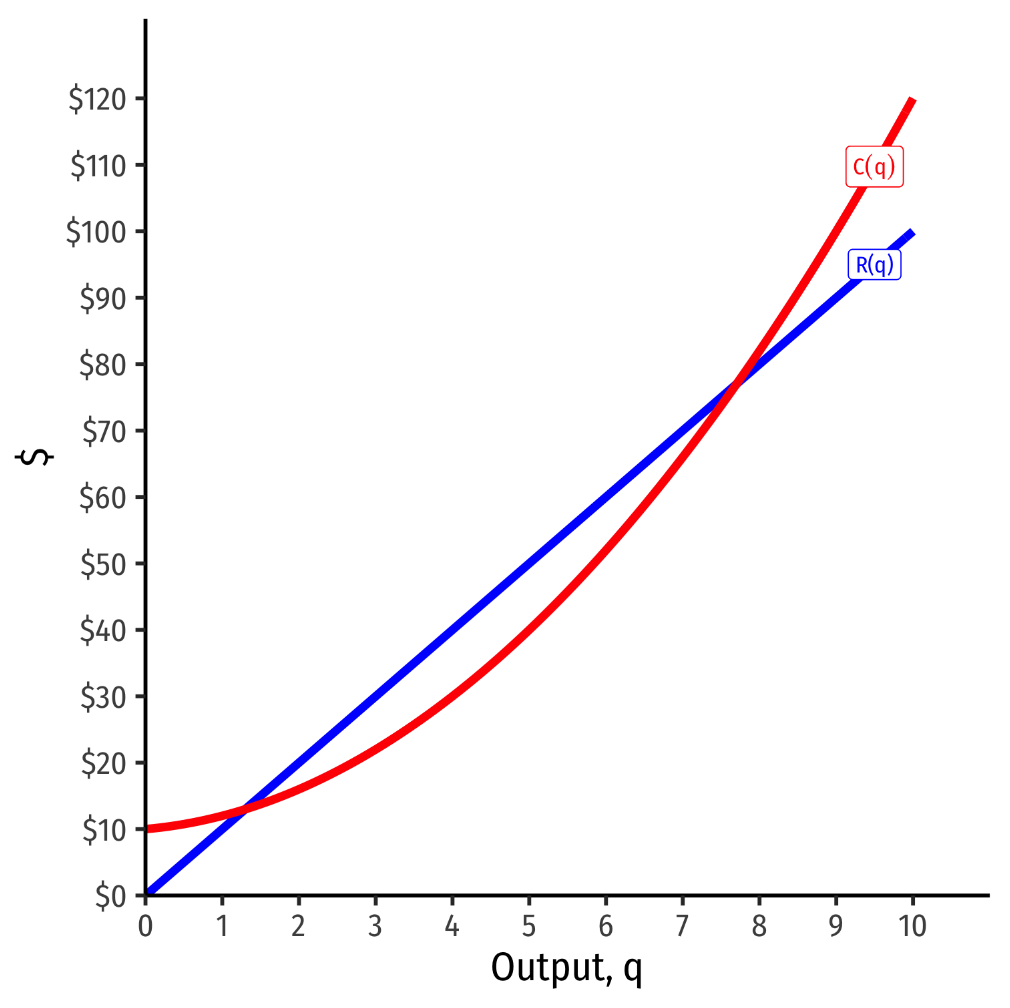
2. **In order to *minimize*:** **< cost >**

3. **Subject to:** **< producing the optimal output >**

- Minimizing costs $\iff$ maximizing profits

Visualizing Total Profit As $R(q) - C(q)$

- $\pi(q) = R(q) - C(q)$



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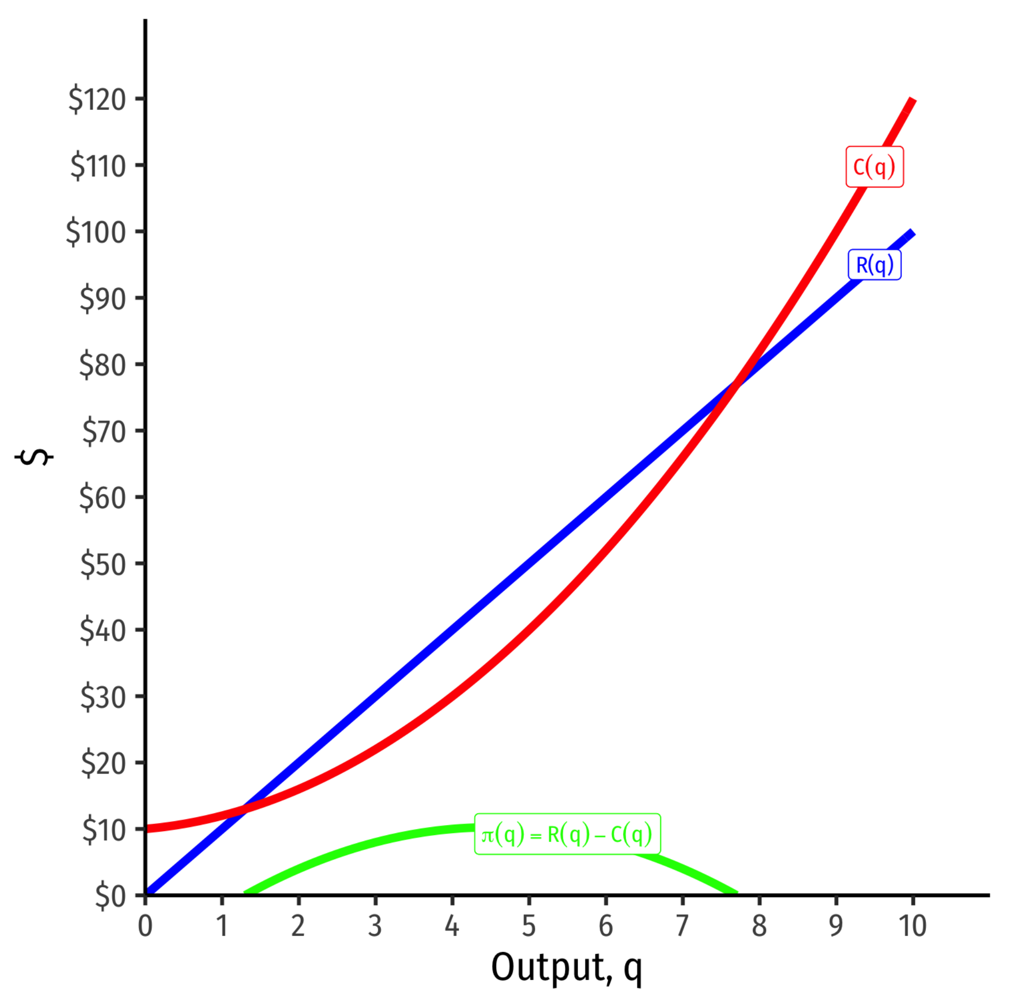
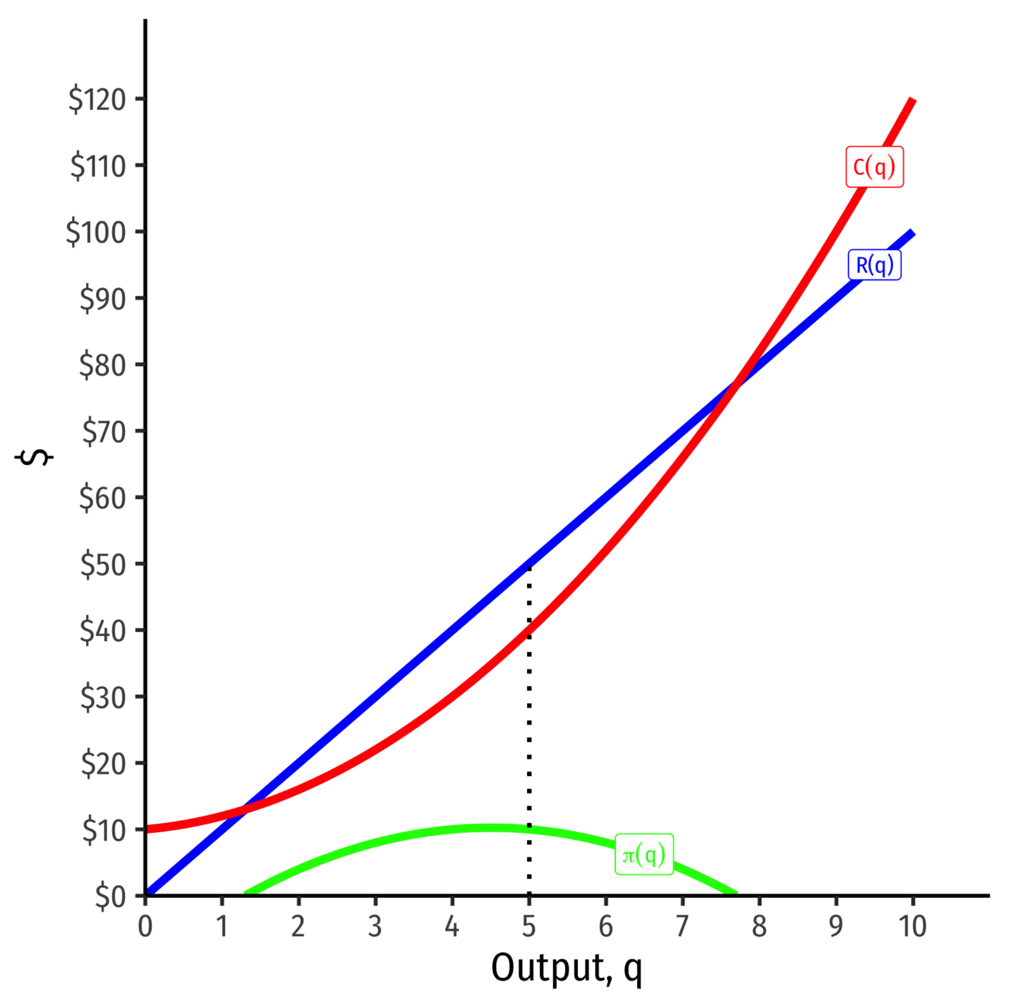


Visualizing Total Profit As $R(q) - C(q)$

- $\pi(q) = R(q) - C(q)$
- Graph: find q^* to max $\pi \implies q^*$ where max distance between $R(q)$ and $C(q)$



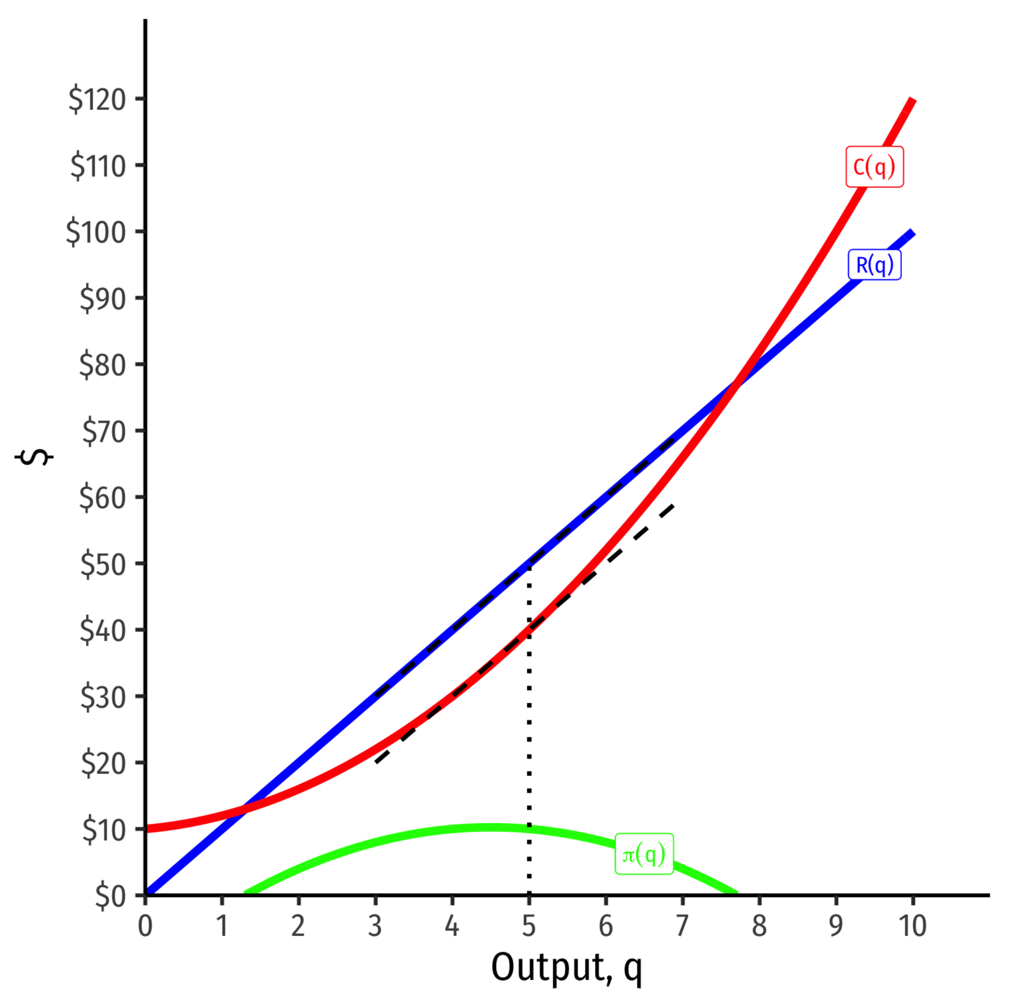
Visualizing Total Profit As $R(q) - C(q)$

- $\pi(q) = R(q) - C(q)$

- Graph: find q^* to max $\pi \implies q^*$ where max distance between $R(q)$ and $C(q)$

- Slopes must be equal:

$$MR(q) = MC(q)$$



Visualizing Total Profit As $R(q) - C(q)$

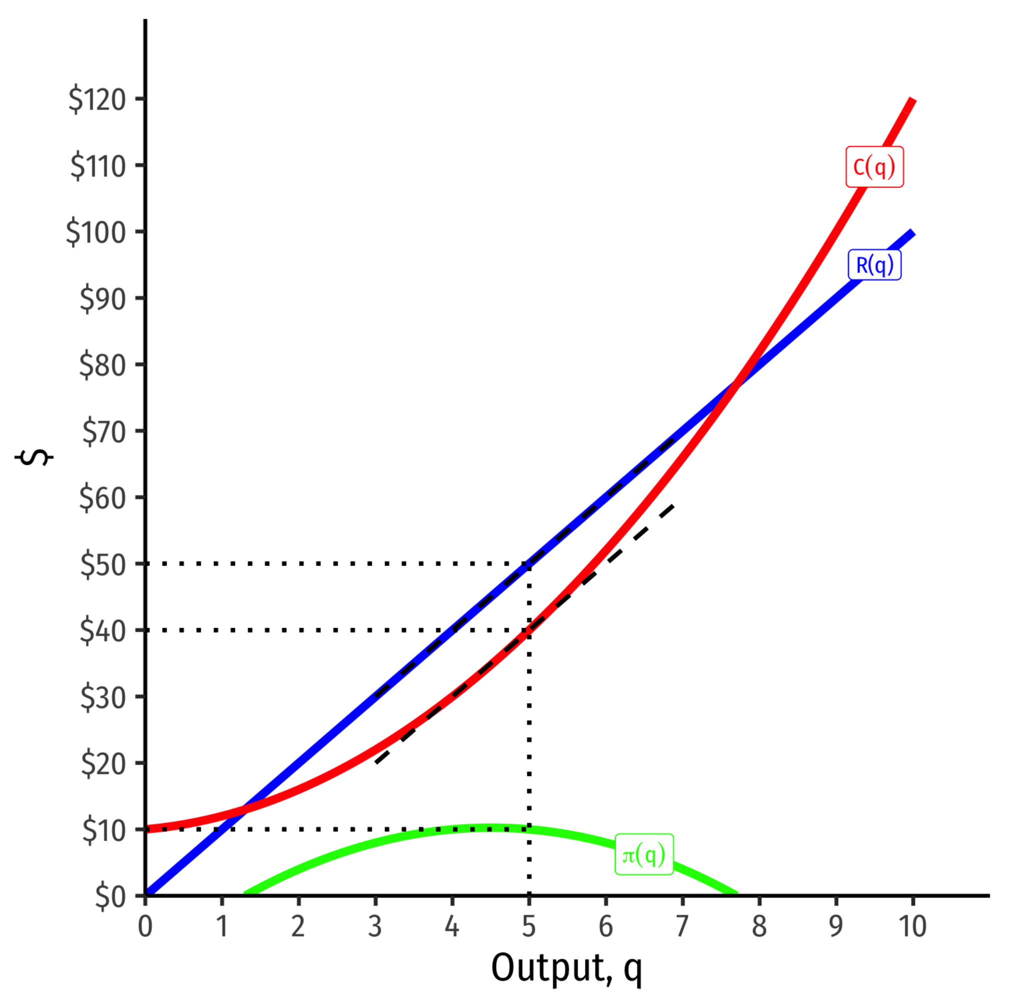
- $\pi(q) = R(q) - C(q)$

- Graph: find q^* to max $\pi \implies q^*$ where max distance between $R(q)$ and $C(q)$

- Slopes must be equal:

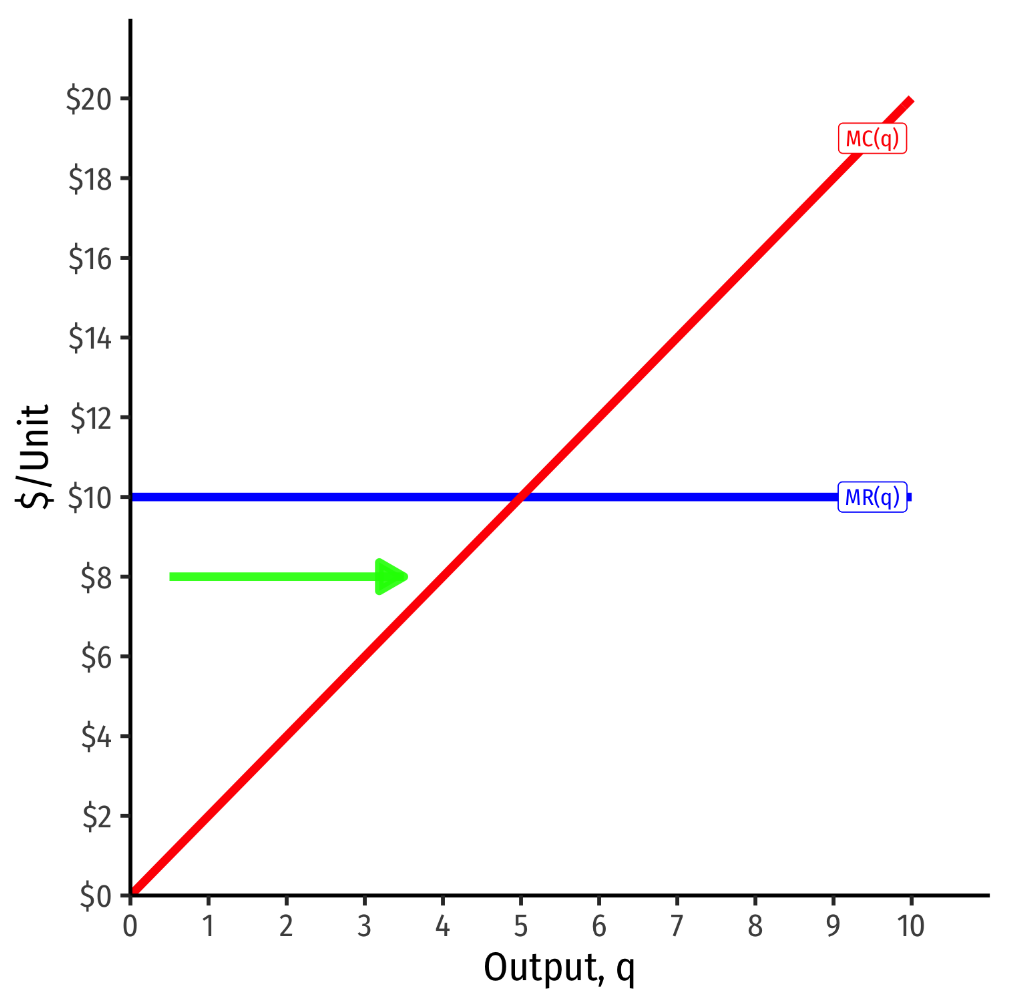
$$MR(q) = MC(q)$$

- At $q^* = 5$:
  - $R(q) = 50$
  - $C(q) = 40$
  - $\pi(q) = 10$



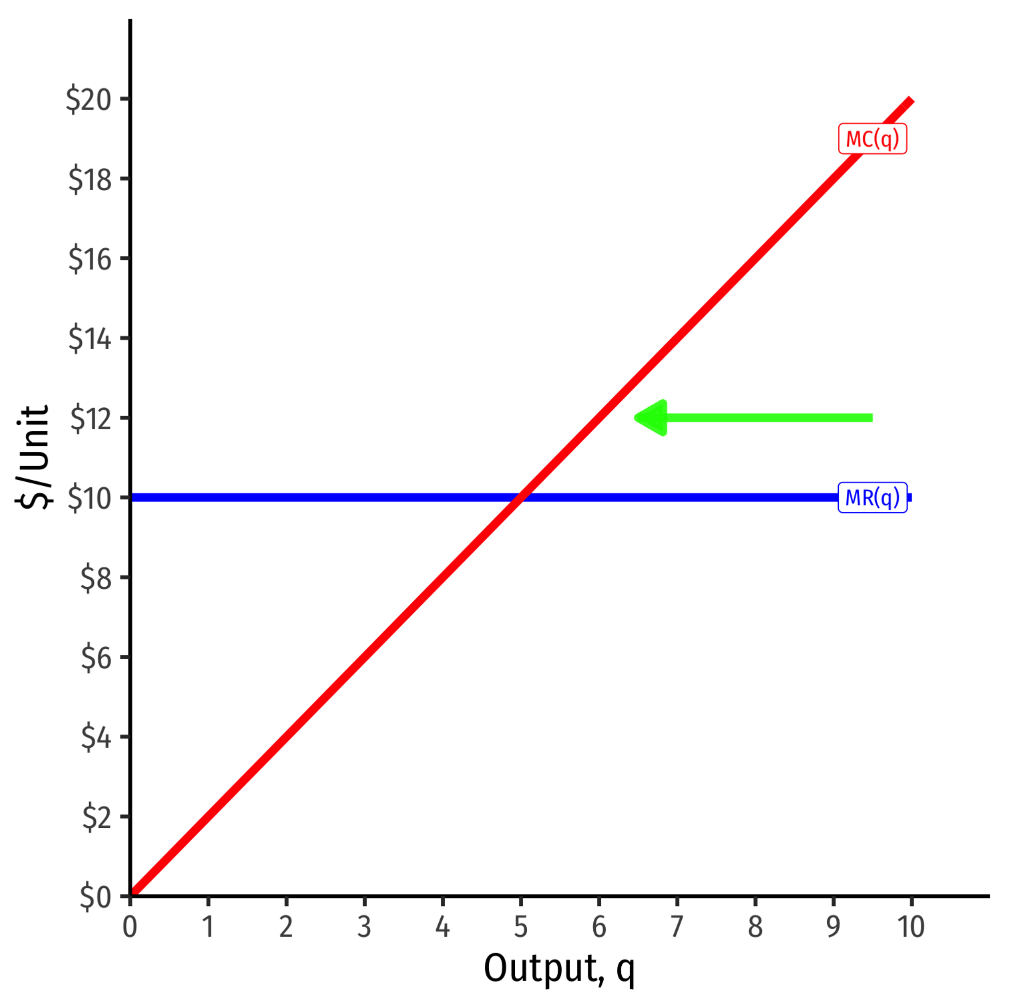
Visualizing Profit Per Unit As $MR(q)$ and $MC(q)$

- At low output $q < q^*$, can increase π by producing *more*: $MR(q) > MC(q)$



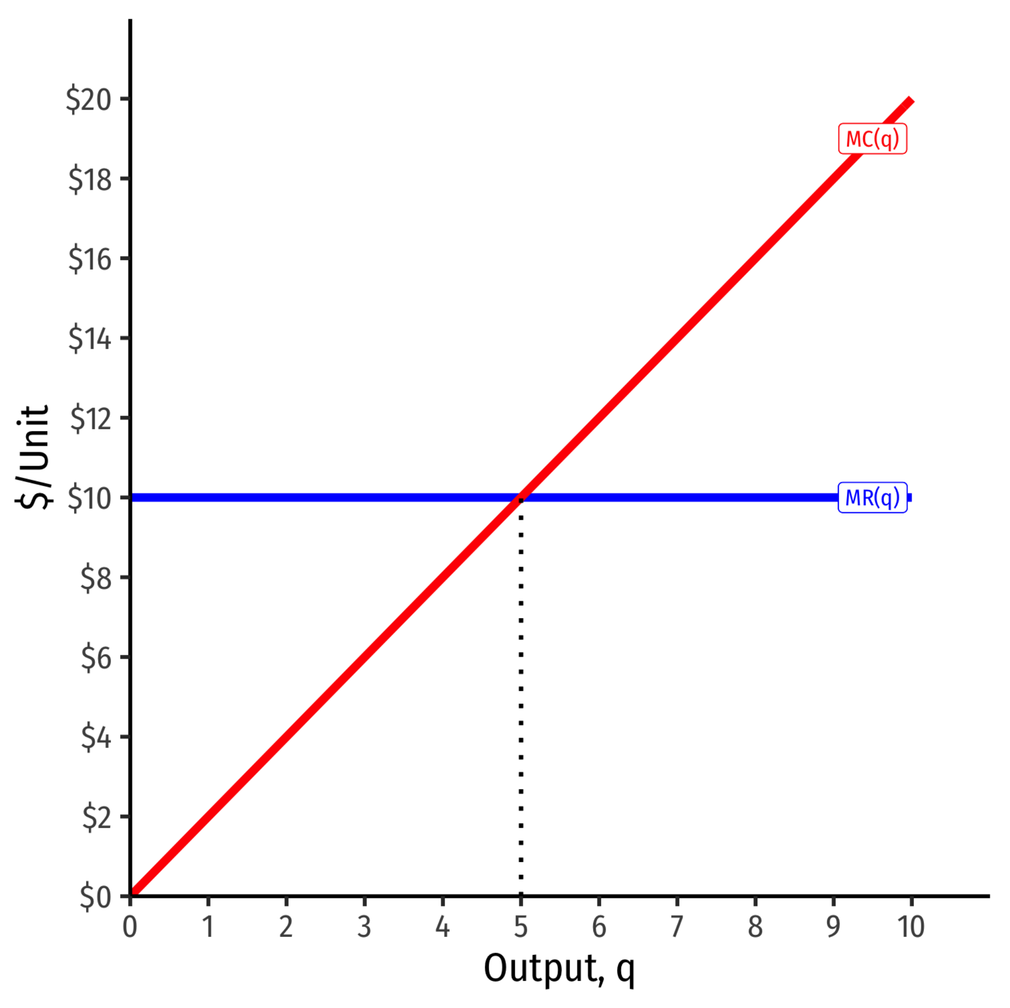
Visualizing Profit Per Unit As $MR(q)$ and $MC(q)$

- At high output $q > q^*$, can increase π by producing *less*: $MR(q) < MC(q)$



Visualizing Profit Per Unit As $MR(q)$ and $MC(q)$

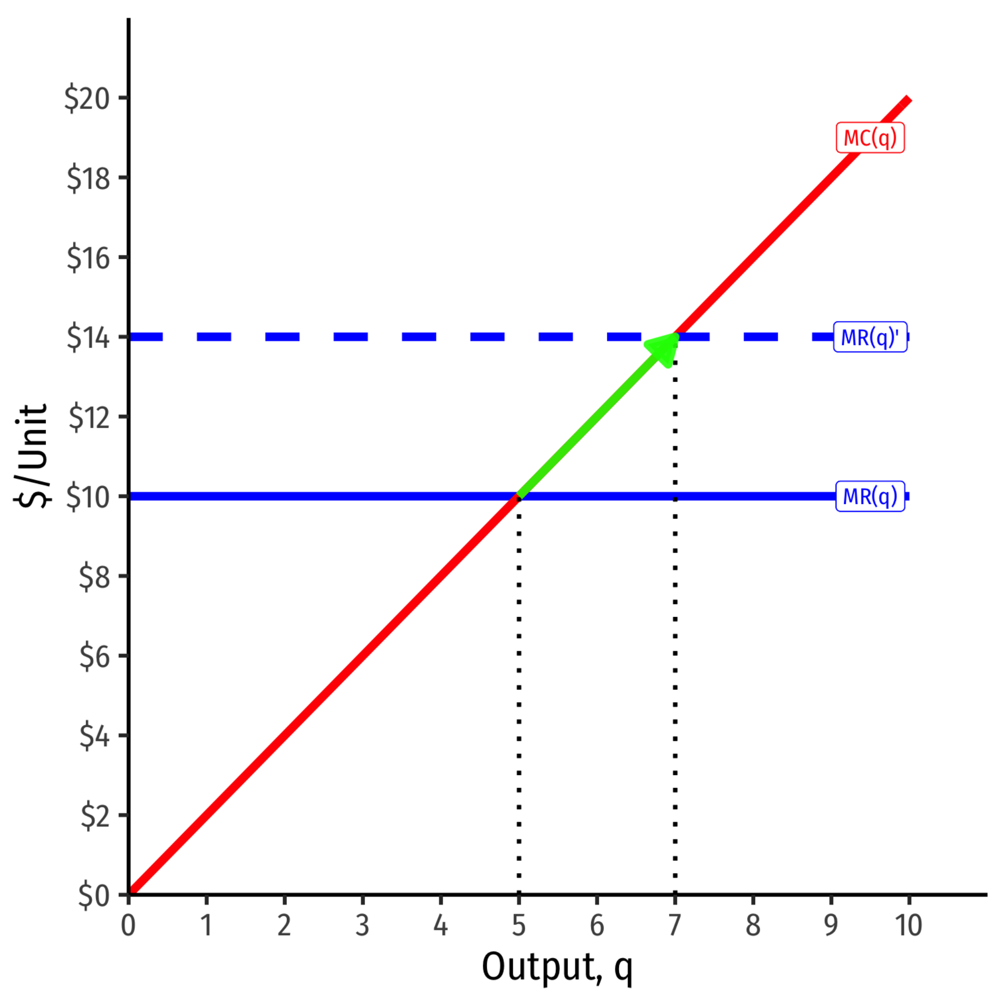
- π is *maximized* where $MR(q) = MC(q)$



Comparative Statics

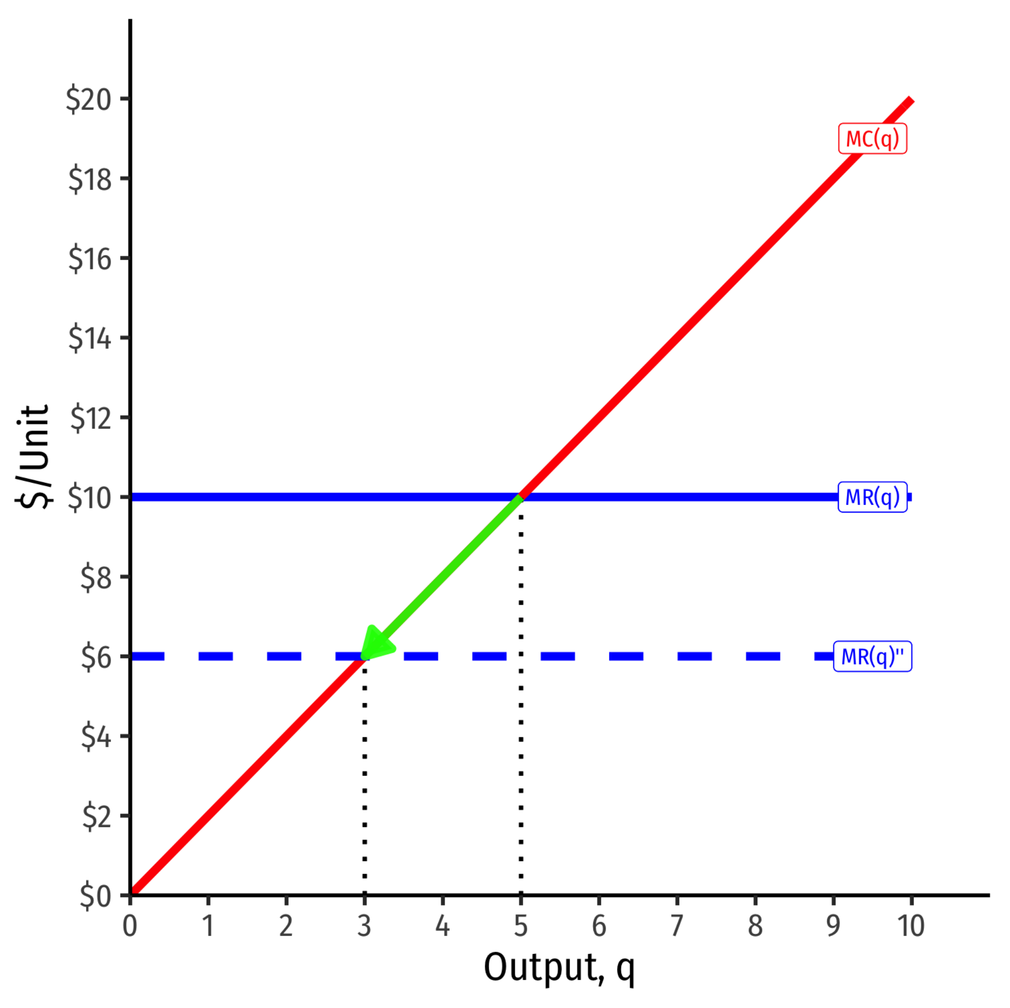
If Market Price Changes I

- Suppose the market price *increases*
- Firm (always setting $MR = MC$) will respond by *producing more*



If Market Price Changes II

- Suppose the market price *decreases*
- Firm (always setting $MR = MC$) will respond by *producing more*



If Market Price Changes II

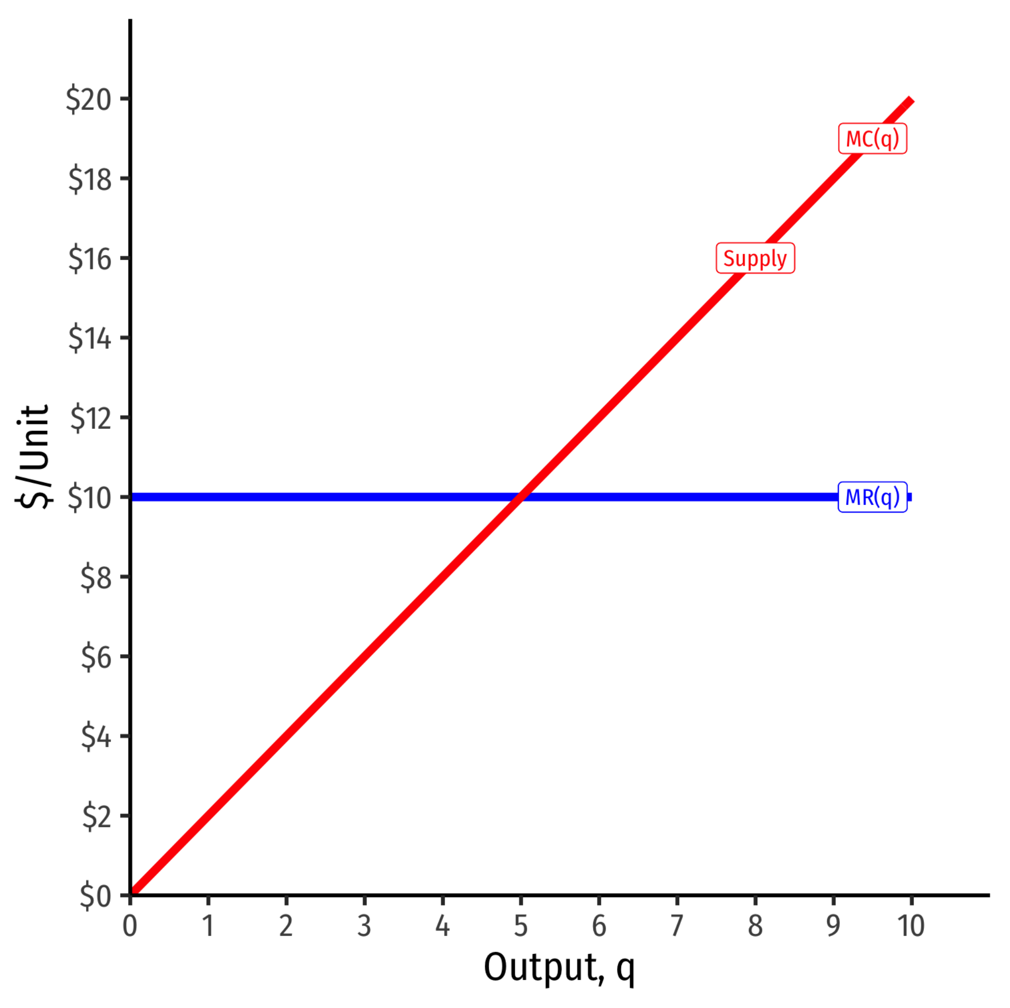
- **The firm's marginal cost curve is its (inverse) supply curve**†

$$Supply = MC(q)$$

  - How it will supply the optimal amount of output in response to the market price

- There is an exception to this! We will see shortly!

† Mostly...there is an exception we will see shortly!

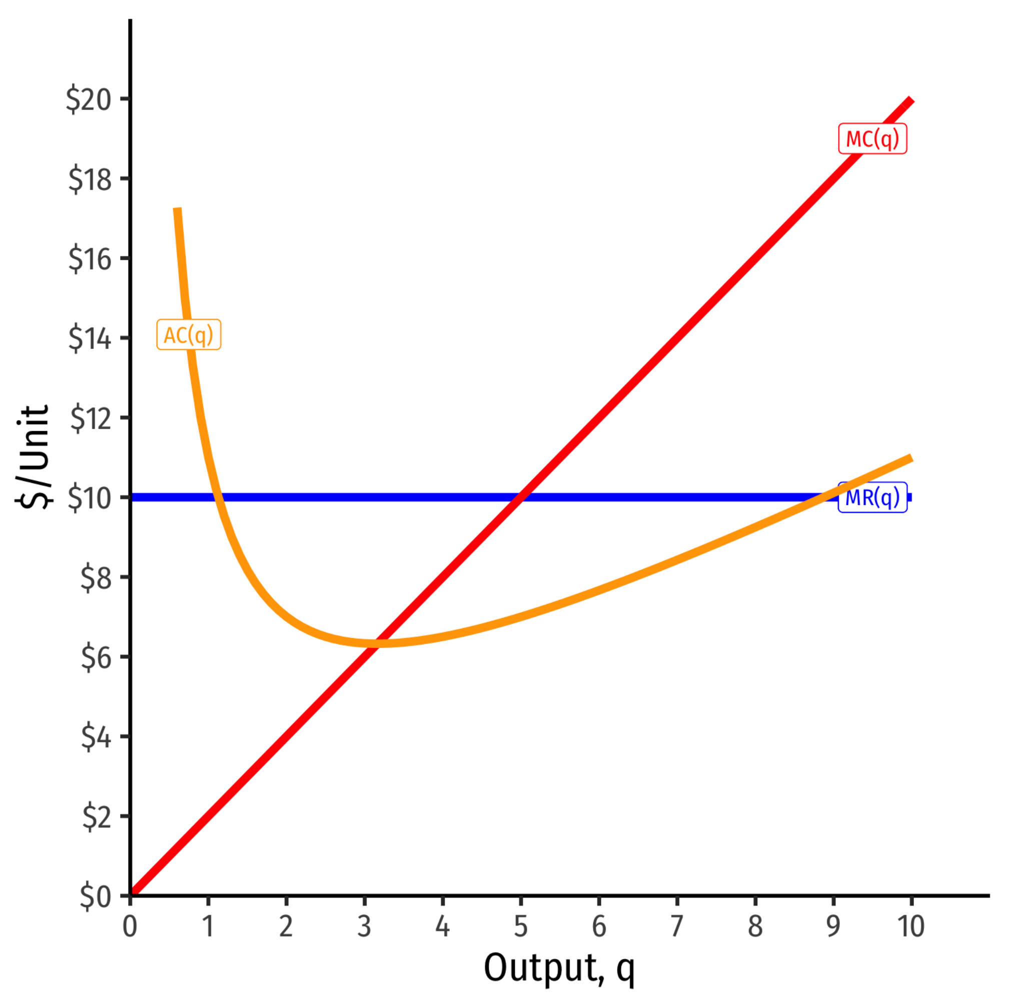


Calculating Profit

Calculating Average Profit as $AR(q) - AC(q)$

- Profit is

$$\pi(q) = R(q) - C(q)$$



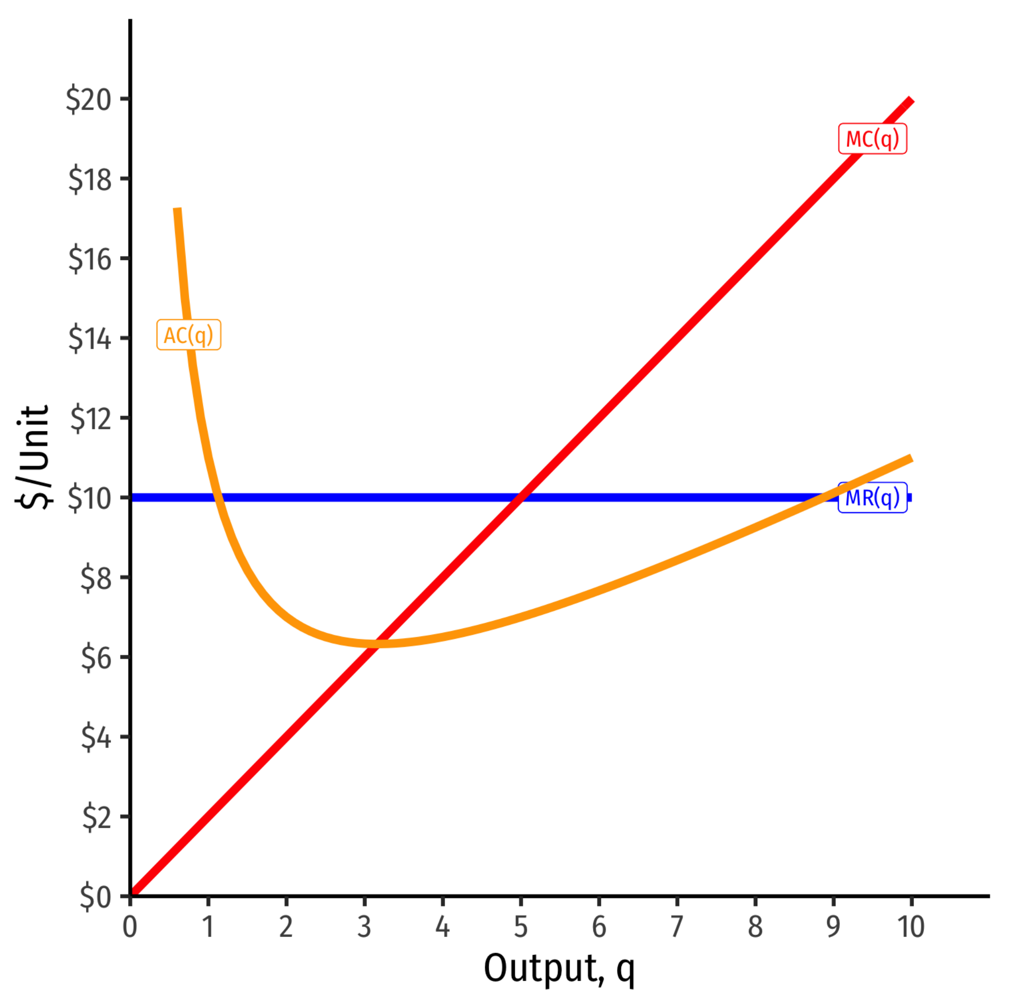
Calculating Average Profit as $AR(q) - AC(q)$

- Profit is

$$\pi(q) = R(q) - C(q)$$

- Profit per unit can be calculated as:

$$\begin{aligned}\frac{\pi(q)}{q} &= \color{blue}{AR(q)} - \color{orange}{AC(q)}\\ &= \color{blue}{p} - \color{orange}{AC(q)}\end{aligned}$$



Calculating Average Profit as $AR(q) - AC(q)$

- Profit is

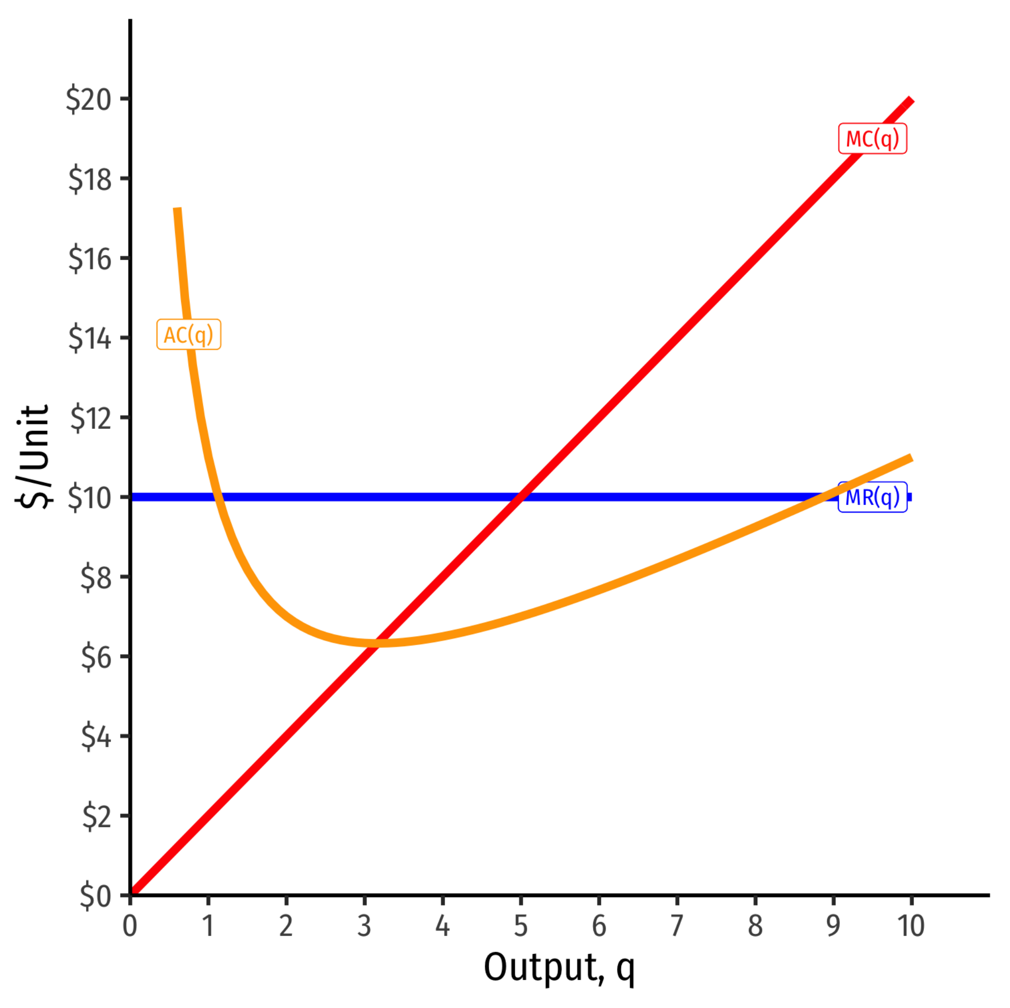
$$\pi(q) = R(q) - C(q)$$

- Profit per unit can be calculated as:

$$\begin{aligned}\frac{\pi(q)}{q} &= {\color{blue}AR(q)} - {\color{orange}AC(q)}\\ &= {\color{blue}p} - {\color{orange}AC(q)}\end{aligned}$$

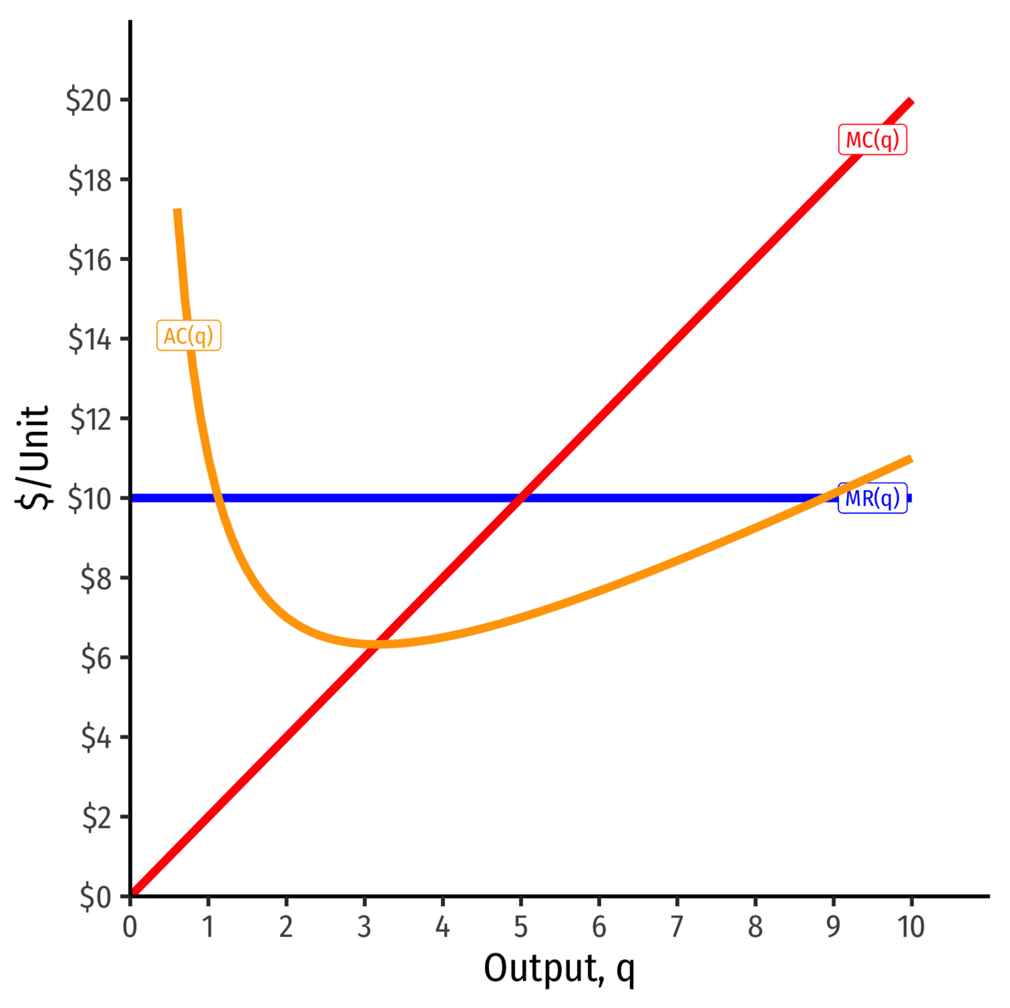
- Multiply by q to get total profit:

$$\pi(q) = q\,[{\color{blue}p} - {\color{orange}AC(q)}]$$



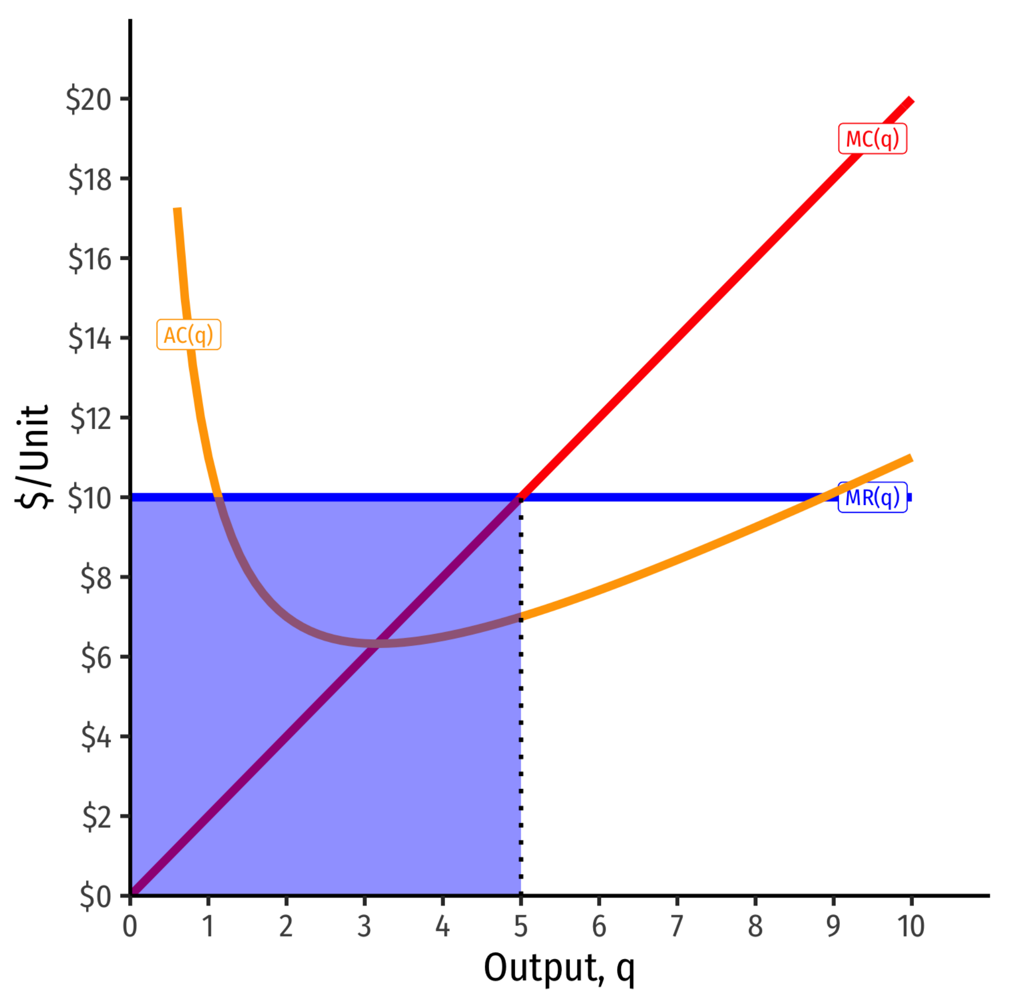
Calculating Average Profit as $AR(q) - AC(q)$

- At market price of p* = $10
- At q* = 5 (per unit):
- At q* = 5 (totals):



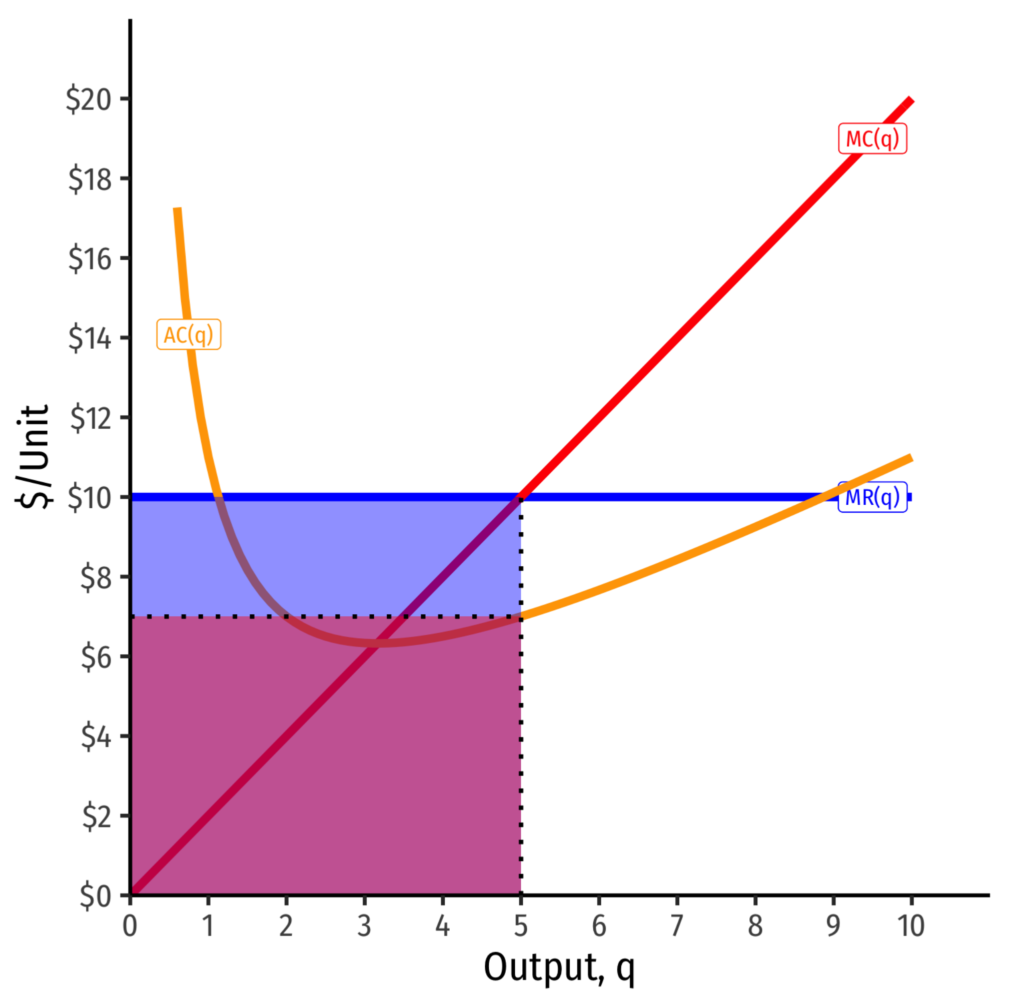
Calculating Average Profit as $AR(q) - AC(q)$

- At market price of p* = $10
- At q* = 5 (per unit):
  - AR(5) = $10/unit
- At q* = 5 (totals):
  - R(5) = $50



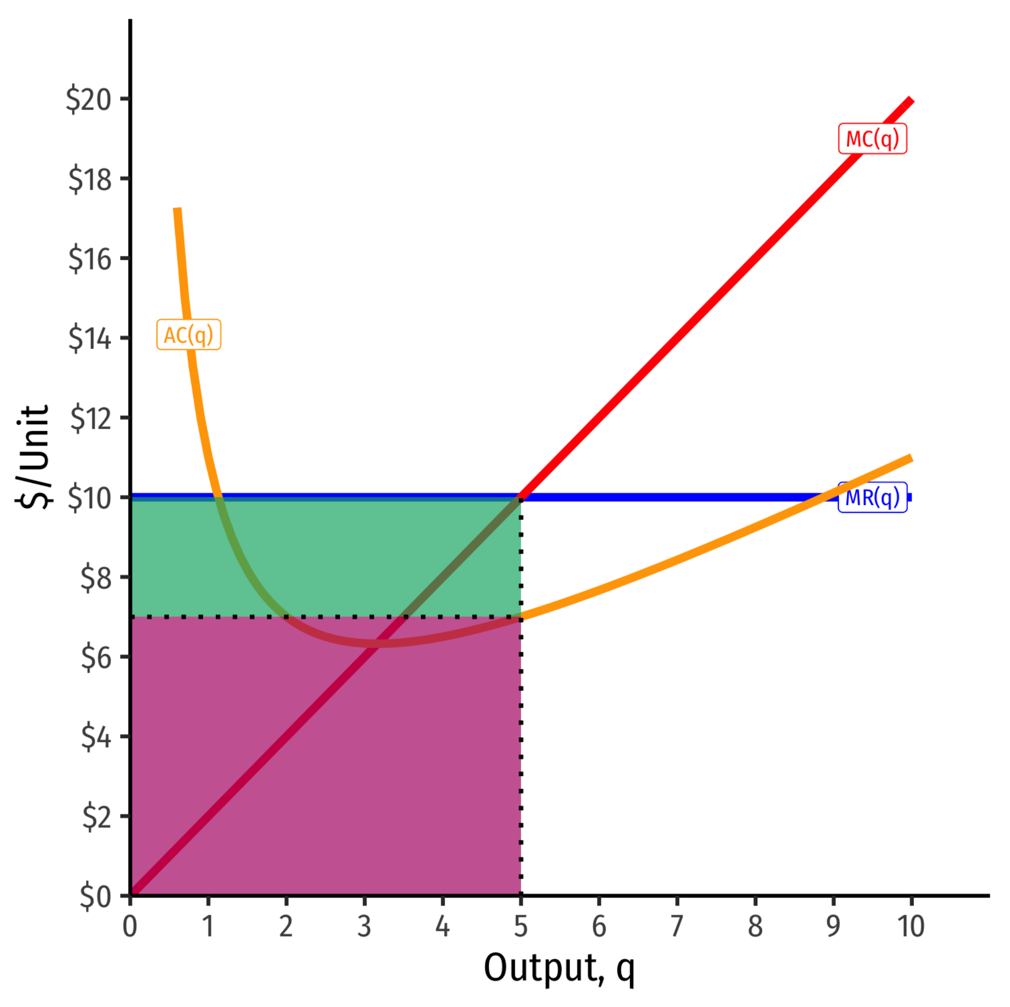
Calculating Average Profit as $AR(q) - AC(q)$

- At market price of $p^* = \$10$
- At $q^* = 5$ (per unit):
  - $AR(5) = \$10/\text{unit}$
  - $AC(5) = \$7/\text{unit}$
- At $q^* = 5$ (totals):
  - $R(5) = \$50$
  - $C(5) = \$35$



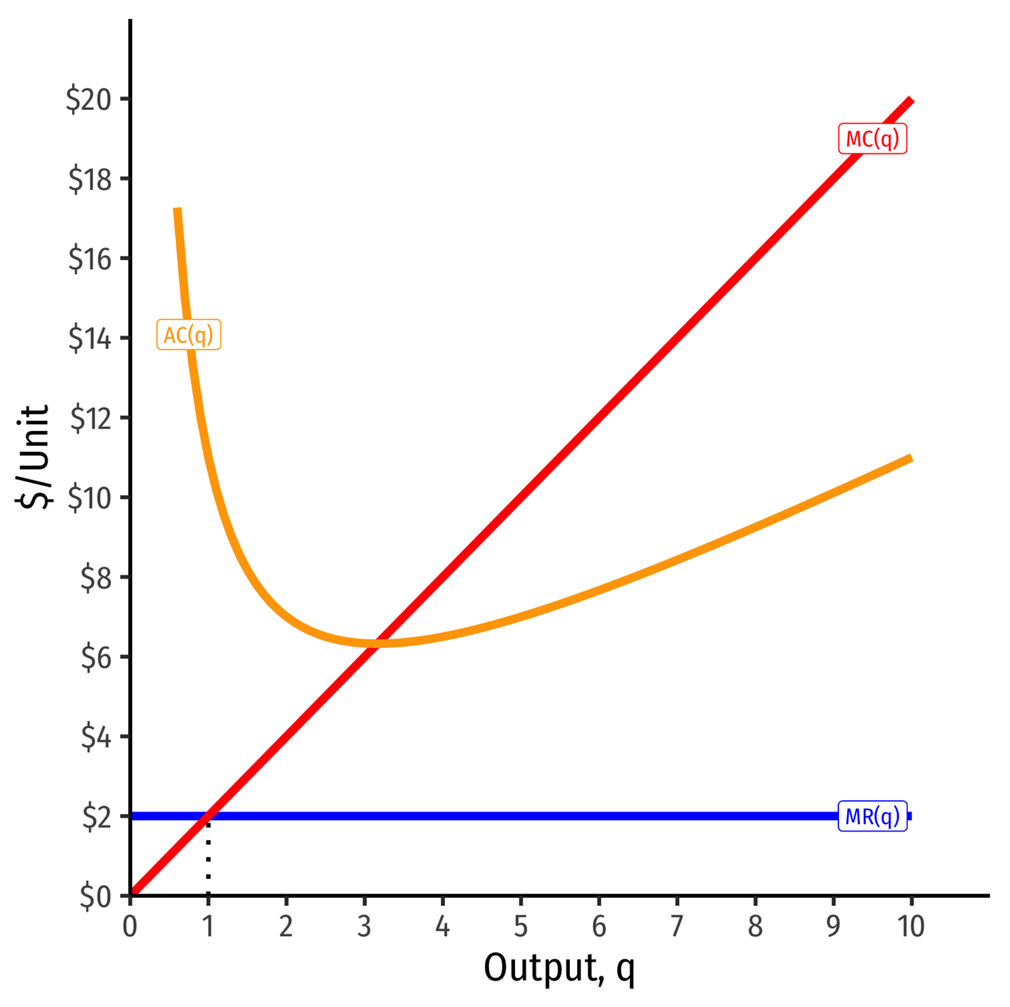
Calculating Average Profit as $AR(q) - AC(q)$

- At market price of p* = $10
- At q* = 5 (per unit):
  - AR(5) = $10/unit
  - AC(5) = $7/unit
  - $A\pi(5)$ = $3/unit
- At q* = 5 (totals):
  - R(5) = $50
  - C(5) = $35
  - π = $15



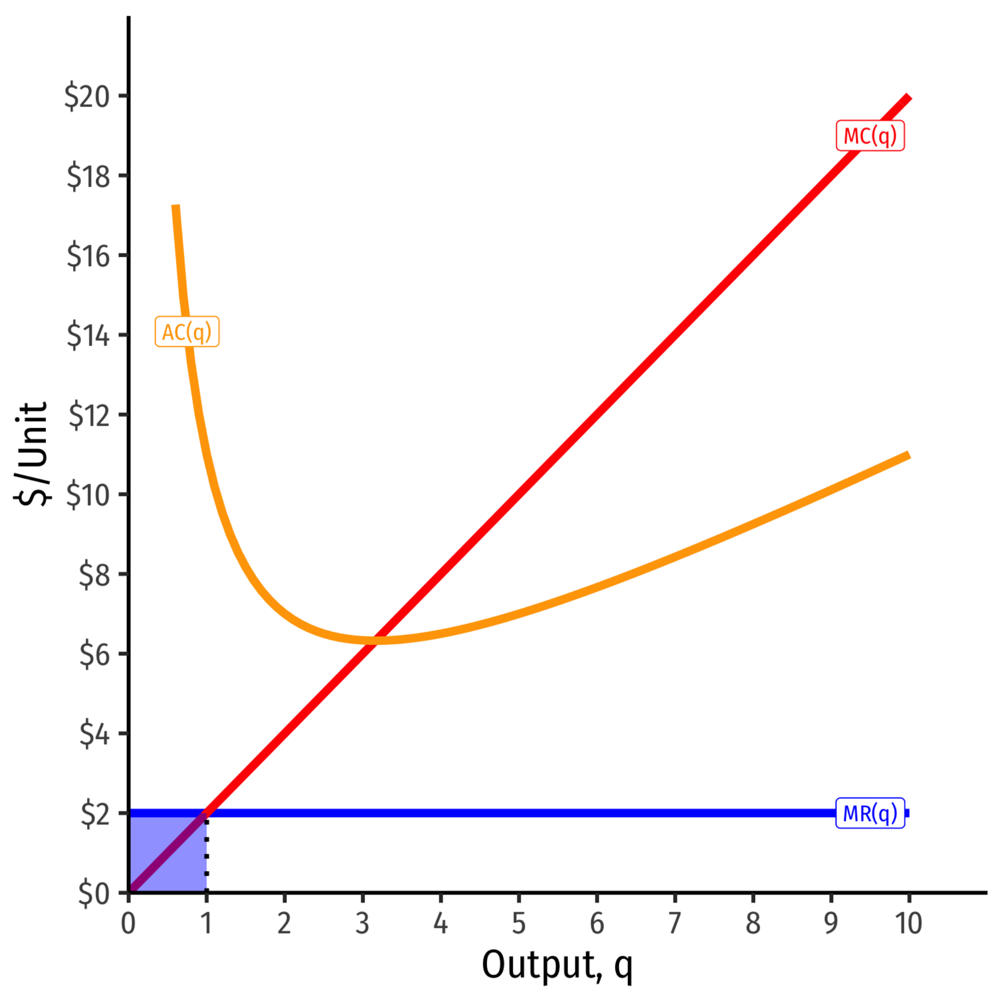
Calculating Average Profit as $AR(q) - AC(q)$

- At market price of p* = $2
- At q* = 1 (per unit):
- At q* = 1 (totals):



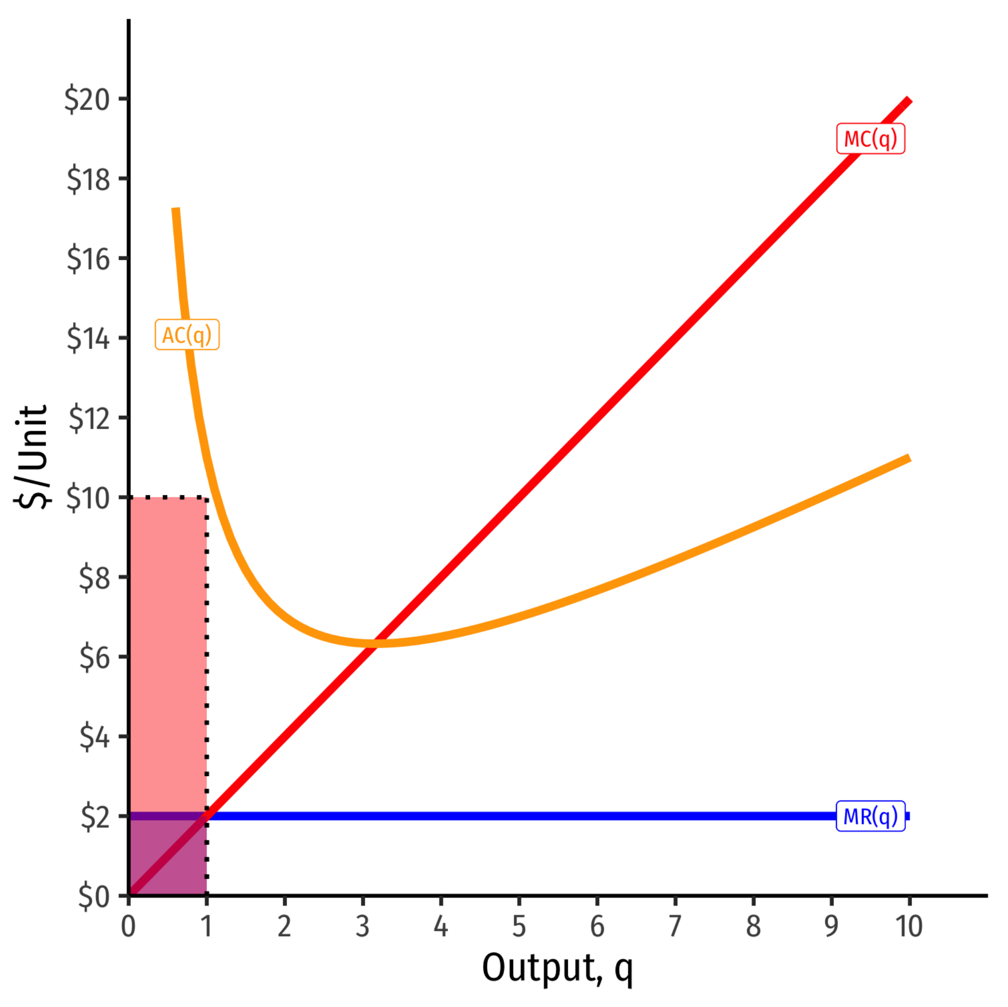
Calculating Average Profit as $AR(q) - AC(q)$

- At market price of p* = \$2
- At q* = 1 (per unit):
  - AR(1) = \$2/unit
- At q* = 1 (totals):
  - R(1) = \$2



Calculating Average Profit as $AR(q) - AC(q)$

- At market price of $p^* = \$2$
- At $q^* = 1$ (per unit):
  - $AR(1) = \$2/\text{unit}$
  - $AC(1) = \$10/\text{unit}$
- At $q^* = 1$ (totals):
  - $R(1) = \$2$
  - $C(1) = \$10$



Calculating Average Profit as $AR(q) - AC(q)$

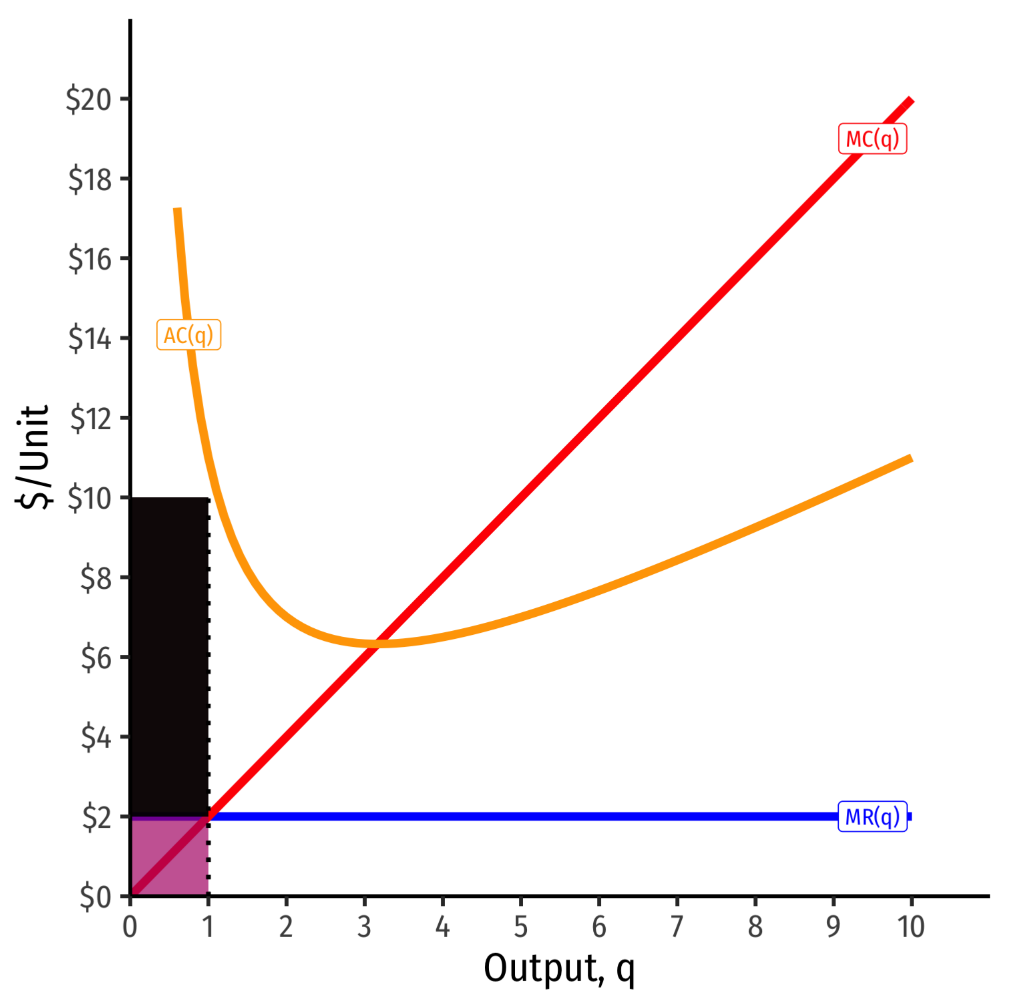
- At market price of $p^* = \$2$

- At $q^* = 1$ (per unit):

  - $AR(1) = \$2/\text{unit}$
  - $AC(1) = \$10/\text{unit}$
  - $A\pi(1) = -\$8/\text{unit}$

- At $q^* = 1$ (totals):

  - $R(1) = \$2$
  - $C(1) = \$10$
  - $\pi(1) = -\$8$



Short-Run Shut-Down Decisions

Short-Run Shut-Down Decisions

- What if a firm's profits at q^* are **negative** (i.e. it earns **losses**)?

- Should it produce at all?

Short-Run Shut-Down Decisions

- Suppose firm chooses to produce **nothing** $(q = 0)$:
- If it has **fixed costs** $(f > 0)$, its profits are:

$$\pi(q) = pq - C(q)$$

Short-Run Shut-Down Decisions

- Suppose firm chooses to produce **nothing** $(q = 0)$:
- If it has **fixed costs** $(f > 0)$, its profits are:

$$\begin{aligned}\pi(q) &= pq - C(q)\\ \pi(q) &= pq - f - VC(q)\end{aligned}$$

Short-Run Shut-Down Decisions

- Suppose firm chooses to produce **nothing** $(q = 0)$:
- If it has **fixed costs** $(f > 0)$, its profits are:

$$
\begin{aligned}
\pi(q) &= pq - C(q) \\
\pi(q) &= pq - f - VC(q) \\
\pi(0) &= -f
\end{aligned}
$$

Short-Run Shut-Down Decisions

- A firm should choose to produce nothing $(q = 0)$ only when:

$$\pi \text{ from producing} < \pi \text{ from not producing}$$

Short-Run Shut-Down Decisions

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$$\pi(q) < -f$$

Short-Run Shut-Down Decisions

- A firm should choose to produce nothing $(q = 0)$ only when:

$$
\begin{aligned}
\pi \text{ from producing} &< \pi \text{ from not producing} \\
\pi(q) &< -f \\
pq - VC(q) - f &< -f
\end{aligned}
$$

Short-Run Shut-Down Decisions

- A firm should choose to produce nothing $(q = 0)$ only when:

$$\begin{aligned}
\pi \text{ from producing} &< \pi \text{ from not producing} \\
\pi(q) &< -f \\
pq - VC(q) - f &< -f \\
pq - VC(q) &< 0
\end{aligned}$$

Short-Run Shut-Down Decisions

- A firm should choose to produce nothing ($q = 0$) only when:

$$\begin{aligned} \pi \text{ from producing} &< \pi \text{ from not producing} \\ \pi(q) &< -f \\ pq - VC(q) - f &< -f \\ pq - VC(q) &< 0 \\ pq &< VC(q) \end{aligned}$$

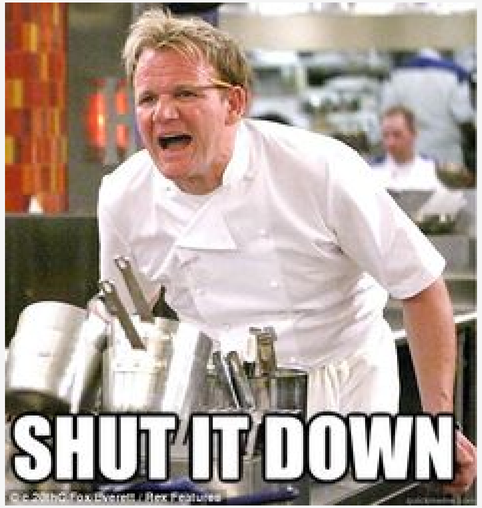
Short-Run Shut-Down Decisions

- A firm should choose to produce nothing $(q = 0)$ only when:

$$\begin{aligned}
\pi \text{ from producing} &< \pi \text{ from not producing} \\
\pi(q) &< -f \\
pq - VC(q) - f &< -f \\
pq - VC(q) &< 0 \\
pq &< VC(q) \\
\mathbf{p} &< \mathbf{AVC(q)}
\end{aligned}$$

Short-Run Shut-Down Decisions

- **Shut down price**: firm will shut down production *in the short run* when $p < AVC(q)$



The Firm's Short Run Supply Decision

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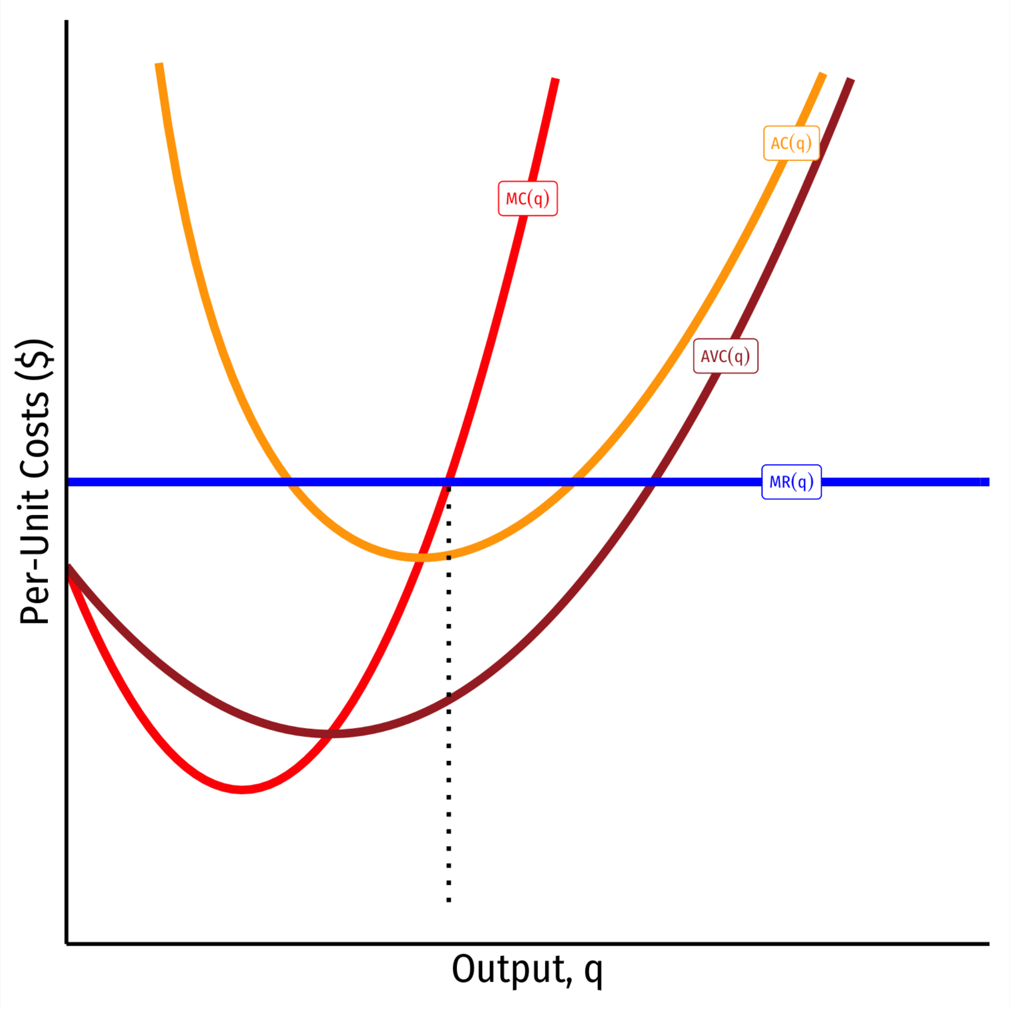
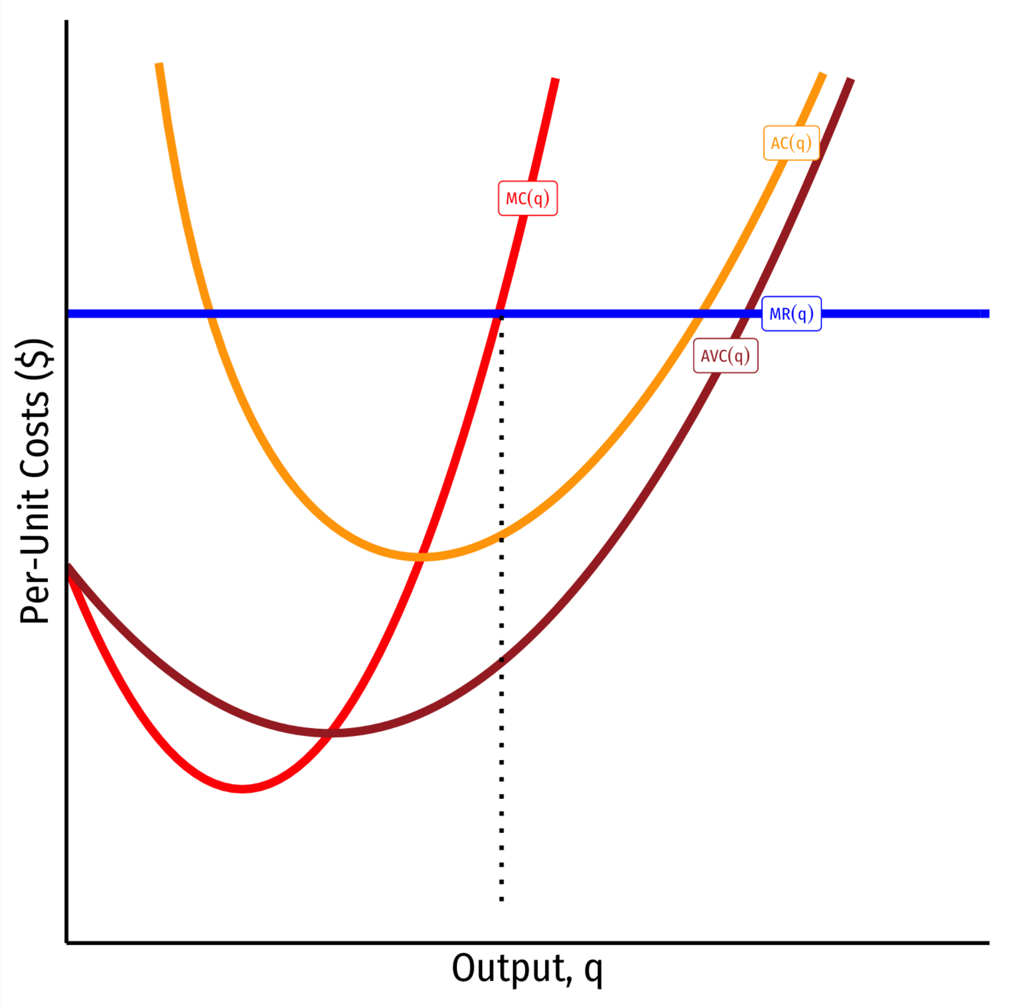
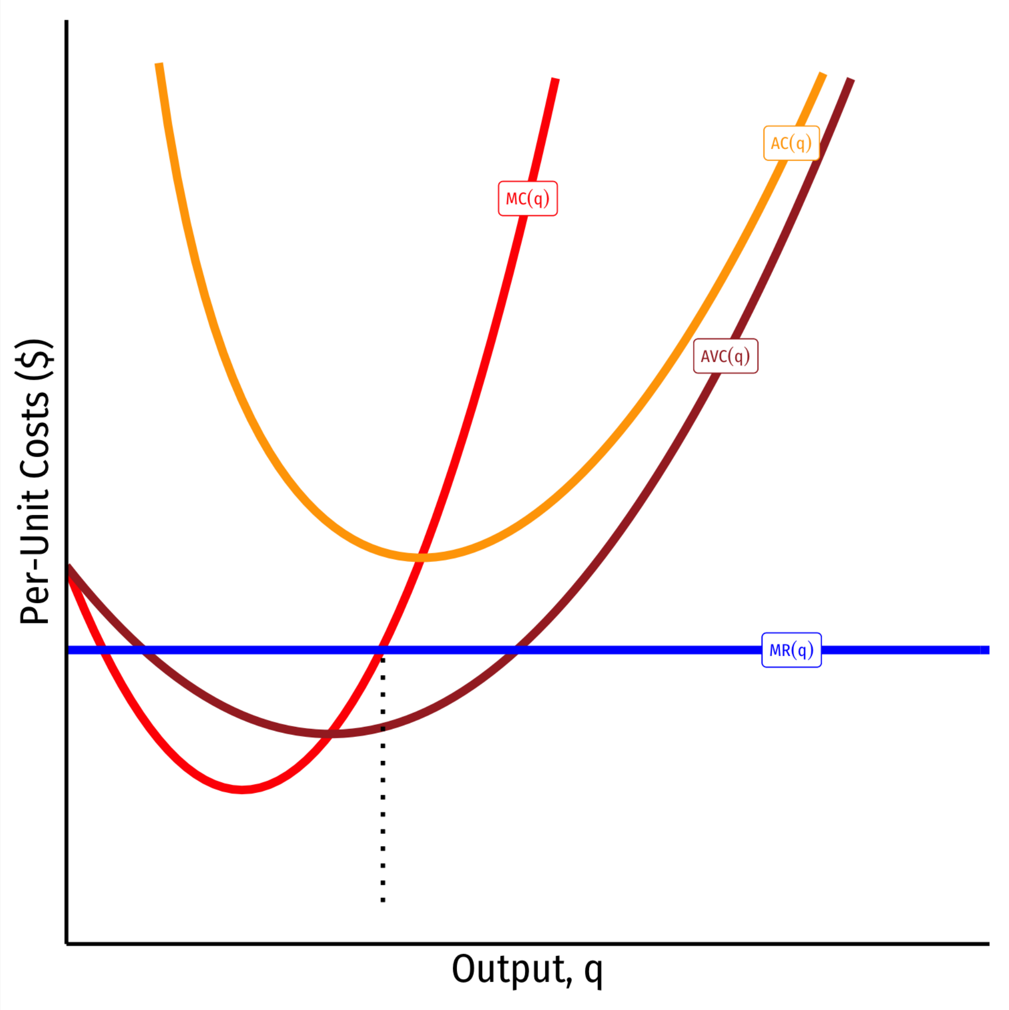


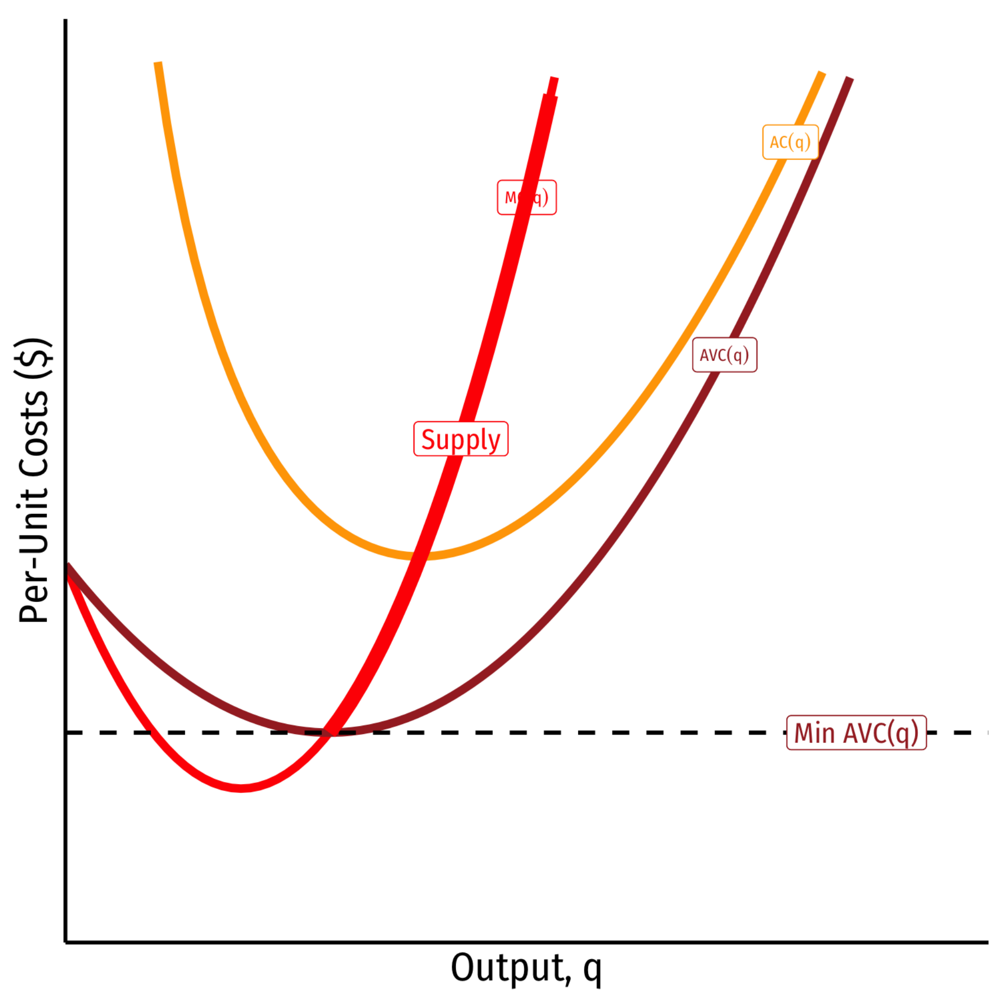
The Firm's Short Run Supply Decision



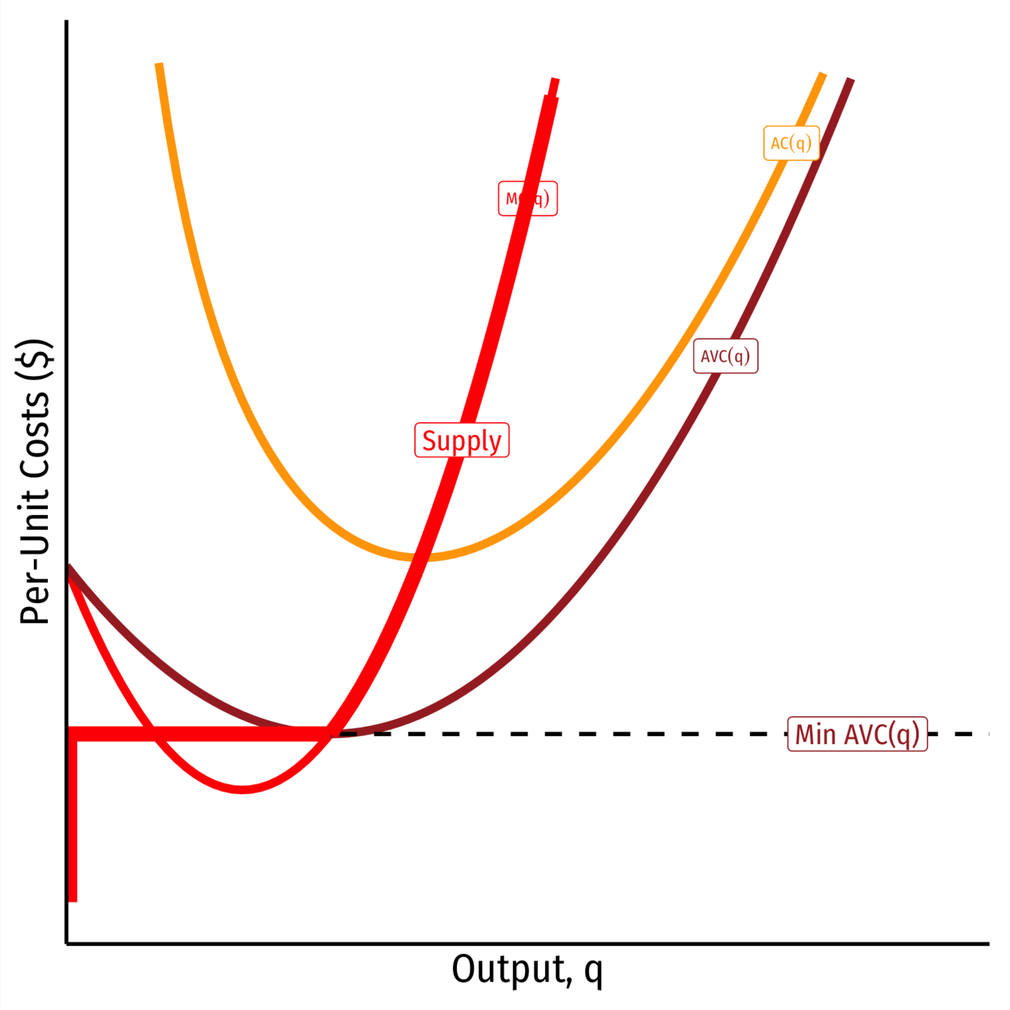
The Firm's Short Run Supply Decision



The Firm's Short Run Supply Decision



The Firm's Short Run Supply Decision



Firm's short run (inverse) supply:

$$\begin{cases} p = MC(q) & \text{if } p \geq AVC \\ q = 0 & \text{If } p < AVC \end{cases}$$

The Firm's Short Run Supply Decision

Firm's short run (inverse) supply:

$$\begin{cases} p = MC(q) & \text{if } p \geq AVC \\ q = 0 & \text{If } p < AVC \end{cases}$$

Summary:

1. **Choose q^* such that $MR(q) = MC(q)$**

2. **Profit $\pi = q[p - AC(q)]$**

3. **Shut down if $p < AVC(q)$**

Firm's short run (inverse) supply:

$$\begin{cases} p = MC(q) & \text{if } p \geq AVC \\ q = 0 & \text{If } p < AVC \end{cases}$$

Choosing the Profit-Maximizing Output q^*: Example

Example: Bob's barbershop gives haircuts in a very competitive market, where barbers cannot differentiate their haircuts. The current market price of a haircut is $15. Bob's daily short run costs are given by:

$$\begin{aligned} C(q) &= 0.5q^2 \\ MC(q) &= q \end{aligned}$$

1. How many haircuts per day would maximize Bob's profits?

2. How much profit will Bob earn per day?

3. Find Bob's shut down price.